Public-Private Catastrophe Risk-Sharing: Theory and Application to COVID-19

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Abstract

Little is known about how an ex-ante public-private risk-sharing program affects the efficiency of a catastrophe risk market and the behavior of market players. We develop a dynamic game model that analyzes three decision makers—individuals, a private insurer, and a government acting as reinsurer—to derive their optimal pricing, capital, and purchasing decisions for efficient catastrophe risk-sharing. In the equilibrium, government reinsurance addresses private insurance market failure and improves social welfare through the product quality and capital cost channels. The effects of these two channels wax and wane depending on the market structure. As a tradeoff, government reinsurance may decrease individuals' expected utilities and increase the insurer's default probability as competition is insufficient in catastrophe insurance markets. In the context of COVID-19, we show that government reinsurance can improve the viability and efficiency of pandemic insurance but should be coupled with anti-monopoly and social-distancing policies to mitigate its downside.

Keywords: catastrophe risk management, efficient risk-sharing, government intervention, public-private partnership, pandemic risk

JEL classification: D81, G22, G52, H84, Q54

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1. Introduction

Unlike an independent risk, a catastrophe risk affects many correlated risk units and causes losses to a large number of correlated individuals and businesses when a catastrophe event occurs. For example, heavy rainfall may cause losses to many farmers in the region where it occurs. The COVID-19 pandemic is another example; it has taken the lives and health of many people and has caused business interruptions worldwide. As a result of the high risk correlation, the aggregate losses of a catastrophe risk are large, heavy-tailed, and more likely to endanger the solvency of insurers even though the loss to each individual might be limited (Charpentier and Le Maux, 2014). This paper develops a theoretical framework for analyzing the optimal public-private catastrophe risk-sharing and applies it to the ongoing COVID-19 pandemic.

Catastrophe (re)insurance facilitates and accelerates post-catastrophe economic recovery (Von Peter et al., 2012). However, the increasing frequency and severity of catastrophe risks endanger the viability of private catastrophe (re)insurance (Froot, 2001). The private (re)insurance market fails to insure (or under-insures) catastrophe risks due to individuals' inadequate willingness to pay (Kousky and Cooke, 2012), high capital costs (Zanjani, 2002), ambiguous risk distribution (Hogarth and Kunreuther, 1985), and non-diversification traps (Ibragimov et al., 2009). In 2017, the insured losses from catastrophes worldwide reached the highest level ever in a single year at USD 144 billion (Swiss Re, 2018). Catastrophe events are expected to increase in both frequency and severity due to epidemics/pandemics, climate change, and cyber attacks.

The COVID-19 pandemic has caused the most severe health and economic crisis in a century. As of November 15, 2020, it has infected 53.8 million people worldwide and killed 1.3 million of them (WHO, 2020). The COVID-19-related business interruption losses in the U.S. are estimated at $1 trillion per month (Hartwig et al., 2020). The uncertainties that surround the COVID-19 pandemic, including the mortality rate and the immunity duration, reduce the optimal initial rate of confinement and thus accelerate the spread of the pandemic (Gollier, 2020). Individuals are willing to pay a price of 24% of the 2019 GDP per person in the U.S. to reduce the probability of COVID-19 infection by 90% (Echazu and Nocetti, 2020). However, the consequences of a pandemic cannot be covered in their entirety by the private insurance market (Richter and Wilson, 2020). Standard business interruption policies typically exclude communicable diseases, which can only be insured via endorsements with limited capacity and restrictive terms (Munich Re, 2020), thus calling for public-private partnership in pandemic risk-sharing (Hartwig et al., 2020).

Governments respond to catastrophe risks by offering two types of catastrophe risk-sharing programs: (i) ex-ante (re)insurance programs and (ii) ex-post relief programs. An ex-ante government (re)insurance program functions to prepare for catastrophe events and create reserves in order to prevent insurer default. An ex-post government relief program bails out catastrophe victims or private insurers after events occur and when private insurers default or are about to default. Both ex-ante and ex-post programs are common in practice. Two government (re)insurance programs are under discussion in the U.S. to support future pandemic risk-sharing: the Pandemic Risk Insurance Act and the Business Continuity Protection Program. In China and Japan, full indemnity of COVID-19 related medical expenses from social insurance and general taxes is offered to qualified COVID-19 patients—an example of government-provided ex-post catastrophe relief. After the 9/11 terrorist attacks, the U.S. Terrorism Risk Insurance Act created a federal-level system to support sharing of certain insured losses resulting from a certified act of terrorism, which is in essence a type of excess-of-loss reinsurance provided by the government. In France, insurance companies purchase catastrophe reinsurance from the Caisse Centrale de Réassurance, a state-owned reinsurer that provides insurers operating in France with coverage against natural catastrophes and uninsurable risks. Charpentier and Le Maux (2014) model the catastrophe risk market with an ex-post catastrophe relief program that covers the default liability of a private insurer. Our paper develops a new model to complement Charpentier and Le Maux (2014) by analyzing the catastrophe risk market with an ex-ante government reinsurance program.

We develop a dynamic game model with three decision makers: a continuum of homogenous individuals, a private insurer, and a government acting as reinsurer. Individuals maximize their expected utilities by choosing whether to buy the catastrophe insurance. The insurer maximizes its expected profit by determining its holding capital, deciding whether to buy the government reinsurance, and (in some markets) setting the catastrophe insurance premium. The government maximizes the social welfare, defined as the expected social utility, by setting the reinsurance premiums (charged to the insurer) and catastrophe taxes (charged to all individuals exposed to the catastrophe risk).

The model enables us to fully characterize the equilibrium of the catastrophe risk market with government reinsurance (GR) and to compare it with the cases of no reinsurance (NR) and private reinsurance (PR). In the GR equilibrium, the insurer is willing to provide full insurance to all individuals. Individuals' demand for the catastrophe insurance (i.e., individuals' maximum willingness to pay for catastrophe risk transfer) is higher than that in the NR and PR equilibriums. Government reinsurance improves social welfare. As a tradeoff, the government reinsurance may decrease individuals' expected utilities and increase the insurer's default probability in catastrophe risk markets where competitions are insufficient.

The three-decision-maker model introduces the government's tradeoff between the use of catastrophe taxes and the use of reinsurance premiums to fund the program and to identify new channels whereby government risk-sharing programs can impact the catastrophe risk market. Reinsurance, whether private or provided by the government, serves the same default-preventing function as an insurer's holding capital but with lower cost (Shiu, 2010). Thus, reinsurance improves social welfare and decreases the insurer's default probability compared to the case of no reinsurance (Bernard and Tian, 2009). Government reinsurance with the tax-premium tradeoff further improves the social welfare over private reinsurance through two channels: (i) the product quality channel: the insurer holds more capital to improve the product quality of the catastrophe insurance, and (ii) the capital cost channel: the individuals increase their maximum willingness to pay for catastrophe risk transfer and thus save the capital cost of the catastrophe insurance. These two channels have opposite impacts on the insurer's default probability: Government reinsurance decreases the insurer's default probability through the product quality channel but increases it through the capital cost channel. Both channels highlight the strategic behavior changes of individuals and the insurer. As competition in a catastrophe insurance market decreases (increases), the insurer has stronger (weaker) market pricing power and is more like a price maker (taker), the product quality channel weakens (strengthens) and the capital cost channel strengthens (weakens). The product quality channel shuts down in a monopolistic catastrophe insurance market, while the capital cost channel shuts down in a perfectly competitive market.

Our model calibrations based on various COVID-19 scenarios show that government reinsurance can improve the efficiency of pandemic risk-sharing compared to the NR and PR cases, and the efficiency gains are larger as the pandemic spreads. However, the efficiency gains from government reinsurance are insufficient to compensate the welfare losses when the pandemic becomes massively widespread. Therefore, any government pandemic (re)insurance program should be coupled with social distancing measures to control the spread of the disease. Moreover, the efficiency gains of government reinsurance are traded off with a "pandemic tax" that decreases the expected utilities of individuals in the pandemic risk market, where competition is typically insufficient.

Our model is extended to incorporate a key difference between the COVID-19 pandemic and many natural catastrophes, i.e., the inter-temporally correlated pandemic losses. Both private and government reinsurance can increase the viability of pandemic insurance and encourages businesses to open, given any level of correlation among individual risks. However, government reinsurance is less effective than private reinsurance to ensure the insurance viability when the inter-temporal correlation is present. When the inter-temporal correlation is excessively large, neither government nor private reinsurance can help to sustain the market. Therefore, any government pandemic reinsurance program should be coupled with business lockdown policies to reduce the inter-temporal correlation of pandemic losses.

Last but not least, our results suggest that the Pandemic Risk Insurance Act discussed in the House (a government reinsurance program) might be more efficient in terms of social welfare and more financially sustainable than the industry-backed Business Continuity Protection Program (a government insurance program) in the low-competition and low-frequency pandemic risk environment. The benefits of the tax-premium tradeoff in a government reinsurance program also inform the optimal design of PRIA—a mix of funding from taxpayers and private insurers.

Contribution and Relation to Literature We contribute to modeling the catastrophe risk markets by jointly analyzing both insurance and reinsurance markets that involve three decision makers in one dynamic game. Connections between these two markets are important to the equilibrium of catastrophe risk-sharing because reinsurance changes the strategic decisions of both individuals and insurers (Lewis and Murdock, 1996). However, existing models consider either (i) the primary catastrophe insurance market where the insurer(s) manage catastrophe risks and interact with individuals (Zanjani, 2002; Boyer and Nyce, 2013; Charpentier and Le Maux, 2014) or (ii) the catastrophe reinsurance market where insurer(s) and reinsurer(s) interact and make decisions given the insurer's catastrophe risk portfolio (Froot, 2001; Ibragimov et al., 2009). Our model captures the incentives and endogenizes the decisions of individuals, a private insurer, and a government reinsurer. It is the first theoretical framework that formalizes the interaction and connection between the insurance and reinsurance markets for catastrophe risks and that reveals the tradeoff between the expected social utility and the insurer's default probability for a government risk-sharing policy.

We also contribute to identifying the impact channels of government risk-sharing programs on the equilibrium of an insurance market. Existing insurance models of public-private risk-sharing either assume away the insurer's default probability (Boulatov and Dieckmann, 2012; Boyer and Nyce, 2013; Lehr, 2014) and/or consider the insurer's holding capital to be exogenous (Charpentier and Le Maux, 2014). However, identifying the channels through which government risk-sharing programs change the strategic behaviors of market players and introduce the tradeoff between the expected social utility and the insurer's default probability, probability requires us to simultaneously endogenize the default probability and the capital decision of the insurer. Our new model identifies for the first time the product quality and capital cost channels of a government risk-sharing program's impact and their wane-and-wax patterns depending on the market structure of catastrophe insurance.[1] In this sense, our three-decision-maker risk-sharing model and its suggested government reinsurance solution contribute to the ongoing discussion on efficient risk-sharing using public-private partnerships (Krueger and Perri, 2011; Charpentier and Le Maux, 2014).

Last but not least, we also contribute to modeling COVID-19 and future pandemic risks. The classic Susceptible-Infected-Recovered (SIR) in the public health literature was developed by Kermack and McKendrick (1927) and has been widely used to investigate the optimal strategies to measure and to control for the pandemic risks, for example, to estimate the course of pandemics (Pindyck, 2020), to measure the pandemic-related economic losses (Hall et al., 2020), and to analyze public health policies including optimal confinement intensity (Gollier, 2020) and infection status testing (Brotherhood, 2020). To our best knowledge, the SIR model has not yet been connected with a model of insurance markets and therefore applied to analyze the optimal pandemic risk-sharing. In this paper, we first-time connect the SIR model with a catastrophe risk-sharing model by formalizing the relationship between the basic reproduction number (R_0) and cumulative infection rate in the former, and the individual risk correlation in the latter. This connection enables us to analyze the pandemic risk management from the public-private risk-sharing perspective. It extends the scope of application of the SIR model to the insurance markets.

The rest of the paper is structured as follows. Section 2 sets up the model. Section 3 reports the equilibrium with government reinsurance and our propositions. Section 4 analyzes the impact channels and pricing of government reinsurance. Section 5 applies the catastrophe risk-sharing

[1] Cummins et al. (2002) and Zanjani (2002) endogenize both the default probability and the capital decisions of an insurer but do not analyze government-provided risk-sharing. Schlütter (2018) finds that premium tax and corporate tax impede the insurer's incentive of holding capital, however, he focuses on the effect of general tax policy on non-life insurers, in which government does not participate in risk-sharing.

model to the COVID-19 pandemic. Section 6 discusses alternative government intervention policies. Section 7 concludes. All proofs are provided in the appendices.

2. Model

A large number of homogenous individuals in a region (normalized to 1) is exposed to the catastrophe risk that may cause a loss l ($l > 0$) to each individual with a probability of p ($0 < p < 1$).[2] In the region, individual risks have a correlation of δ ($0 < \delta < 1$). Once the catastrophe event occurs, an x ($0 \leq x \leq 1$) share of the unit mass of individuals suffers losses. Thus, the cumulative distribution of x, $F(x|p,\delta)$, is determined by the probability p that an individual suffers a loss and by the correlation among individual risks δ. The catastrophe-hit share of population x satisfies

$$F(x|p,\delta) = \int_0^x f(z)dz, \int_0^1 xf(x)dx = p. \tag{1}$$

The probability p is equal to the expectation of the catastrophe-hit share of population x. The risk correlation δ determines the shape of x's distribution. The higher the correlation δ is, the more heavy-tailed the x's distribution is, and the more catastrophic the risk is. The individuals and the insurer are assumed to have symmetrical information on the exogenous risk distribution.[3]

Next, we develop our three-decision-maker model based on the above Charpentier and Le Maux's (2014) catastrophe risk definition. Figure 1 illustrates the timeline of the dynamic game. The government offers reinsurance coverage by charging a catastrophe tax T ($T \geq 0$) to each individual in the region and a reinsurance premium M ($M \geq 0$) to the insurer. Observing the catastrophe tax and the reinsurance premium, the insurer decides whether to buy the reinsurance $R \in \{0,1\}$, determines its own holding capital C ($C \geq 0$),[4] and sets the primary insurance premium α ($\alpha > 0$).[5] Next, observing the decisions of the government and the insurer, all homogeneous

[2] Individuals refer to all inhabitants and businesses in a region that are exposed to the catastrophe risk (Charpentier and Le Maux, 2014), e.g., all inhabitants and businesses in one flood zone or in one COVID-19 affected region.

[3] This assumption excludes adverse selection and moral hazard and is consistent with existing literature in that the catastrophe risks are not private information (Jaffee and Russell, 1997; Ibragimov et al., 2009; Charpienter and Le Maux, 2014). In practice, scientific forecasts and guidance for natural disasters are publicly available to both individuals and insurers. For example, the U.K. Environment Agency publishes flood risk maps, which are suitable to guide both individuals and insurers. Some other catastrophe risks, such as terrorism attacks and the COVID-19 pandemic, are symmetrically uninformed to and gradually learned by both individuals and insurers. We allow for and analyze the moral hazard problem in Sections 5.2 and 5.3.

[4] The financial regulatory authority may impose a minimum capital requirement as a constraint on the insurer and thus on the market equilibrium. See Section 6.2 for a detailed discussion on solvency regulation.

[5] Depending on the market structure and the price regulation if any, the insurer can be a price maker, a price taker, or in between with certain market pricing power. Our conclusions hold across market structures and in a wide range of insurance prices between the insurer's minimum acceptable premium and individuals' maximum willingness to pay.

individuals simultaneously decide whether to buy the catastrophe insurance. Individuals make the purchasing decisions based on the catastrophe insurance price and its product quality (i.e., the catastrophe insurer's default probability).[6] The equilibrium of this dynamic game is derived by backward induction starting from the last decision maker—the individuals.

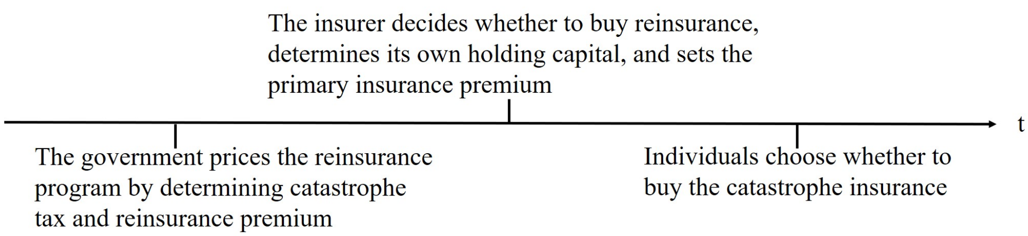

Figure 1: Timeline of the Dynamic Game

2.1. Individuals' Decision

All individuals are assumed to have the same utility function $U(\cdot)$ that is twice differentiable, strictly increasing, and concave. The utility is without initial wealth, depends on the wealth loss, and is always negative.[7] Each individual receives the indemnity $I(x)$ if she bought the insurance. Thus, an individual's expected utility is

$$U_i(\alpha,T) \equiv \begin{cases} \int_0^1 xU(-\alpha - T - l + I(x)) f(x)dx + \int_0^1 (1-x)U(-\alpha - T) f(x)dx, & \text{if buying insurance,} \\ pU(-l-T) + (1-p)U(-T), & \text{if not buying insurance.} \end{cases} \tag{2}$$

An individual buys the catastrophe insurance if and only if her expected utility of buying is equal to or greater than her expected utility of not buying.[8] *Ceteris paribus*, a higher insurance premium decreases an individual's expected utility if she buys the catastrophe insurance. Thus, there exists a maximum willingness to pay $\alpha^*(C,T)$ for each individual such that she is indifferent between buying and not buying catastrophe insurance at this price.[9]

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[6]The insurer's financial strength is a key determinant of insurance purchasing decisions and in particular for catastrophe risk coverage (Zanjani, 2002). The financial strength is determined by an insurer's holding capital and the reinsurance coverage. The information on an insurer's financial strength is usually publicly available and easy to interpret as a rating or solvency ratio in solvency reports. Empirically, individuals are willing to pay a higher price for a safer product with lower insolvency risks (Cummins and Danzon, 1997).

[7]If there is no loss, individuals achieve the highest utility of zero, $U(0) = 0$. This type of utility without initial wealth is commonly used in catastrophe research (see, e.g., Ibragimov et al., 2009).

[8]Without loss of generality, we assume that individuals buy insurance when buying and not buying are indifferent. If individuals do not buy in the border case, the catastrophe insurance premium in the equilibrium will be individuals' maximum willingness to pay minus a sufficiently small positive number ε. All our conclusions hold.

[9]The $\alpha^*(C,T)$ captures the positive relationship between a safer insurance product (i.e., lower default probability with more holding capital) and higher maximum willingness to pay of individuals (Cummins and Danzon, 1997).

2.2. Insurer's Decisions

The insurer offers a full coverage for the catastrophe risk.[10] Its expected profit is as follows:

$$\Pi(\alpha,C,R,M) \equiv \begin{cases} \int_0^{\bar{x}} [(\lambda\alpha - RM) - (\lambda xl - RI_{re}(x))] f(x)dx - [1-F(\bar{x})]C - rC, & if\, \alpha \le \alpha^*(C,T), \\ 0, & if\, \alpha > \alpha^*(C,T), \end{cases} \tag{3}$$

where λ ($0 \le \lambda \le 1$) is the share of individuals that purchases the catastrophe insurance, $I_{re}(x)$ is the reinsurance indemnity, and $\bar{x}$ is the default threshold of the catastrophe-hit share of individuals, above which the insurer defaults.

$$\bar{x} \equiv \frac{1}{\lambda l}[\lambda\alpha + C + R(I_{re}(\bar{x}) - M)]. \tag{4}$$

The first term of Eq.(3) is the insurer's expected profit when it is solvent ($0 < x \le \bar{x}$), which is equal to its retained premium minus its retained loss. The second term is the insurer's expected loss when it is insolvent ($x > \bar{x}$); that is, the insurer loses all its holding capital C with the probability of insolvency $1 - F(\bar{x})$. The last term is the insurer's cost of capital rC, a cost the insurer has to bear no matter it defaults or not, which reflects the expected return of the insurer's shareholders. The insurer is better off with a higher insurance premium and less holding capital. The insurer earns zero profit if it sets the catastrophe insurance premium α above an individual's maximum willingness to pay $\alpha^*(C,T)$, which is essentially equivalent to leaving the market.

The insurer is willing to enter the catastrophe insurance market if and only if it earns a non-negative expected profit from this business. Thus, there exists a minimum acceptable premium $\underline{\alpha}(C,M)$ for the insurer such that its expected profit is zero.[11]

The insurer's profit optimization problem depends on the market structure of the catastrophe insurance, i.e., the insurer's market pricing power. In a monopolistic market, the insurer is a price maker. It maximizes its expected profit by determining the catastrophe insurance price α, its own capital C, and whether to buy the reinsurance R: $\max_{\alpha,C,R}\ \Pi(\alpha,C,R,M)$, s.t. $\alpha > 0$, $C \ge 0$, $R \in \{0,1\}$. In a perfectly competitive market, the insurer is a price taker. Given an exogenous market price $\widetilde{\alpha}$ ($\widetilde{\alpha} > 0$), it decides its capital C and whether to buy the reinsurance R such that its expected

[10] In practice, property insurance without deductible or co-insurance is available in the market, such as, the earthquake insurance in Japan and the Spanish catastrophe program for natural disasters and political-social events (Consorcio de Compensación de Seguros).

[11] The insurer is assumed to be risk-neutral (Biais et al., 2010; Einav et al., 2010). Thus, its expected utility is equal to its expected profit. A risk-neutral insurer may still demand reinsurance to release/save capital, to acquire technical assistance or tax advantages, to meet regulatory requirements, and to avoid bankruptcy cost (Hoerger et al., 1990; Huang and Tzeng, 2007; Bernard and Tian, 2009). Similarly, the government is also assumed risk-neutral in the paper.

profit is zero, $\Pi(\widetilde{\alpha}, C, R, M) = 0$. In markets with imperfect competition, we define the catastrophe insurer's market pricing power $0 \leq \eta \leq 1$ to capture a continuous change in the insurance price as the market structure moves from monopoly ($\eta = 1$) to perfect competition ($\eta = 0$). Thus, in any markets with imperfect competition, the insurer sets the catastrophe insurance premium at a linear combination of individuals' maximum willingness to pay (i.e., the price in a monopolistic market) and the exogenous market price in a perfectly competitive market $\alpha(\eta) = \eta\alpha^*(C,T) + (1-\eta)\widetilde{\alpha}$ and faces a profitability constraint $\Pi(\alpha(\eta), C, R, M) = \eta\Pi^*(M,T)$, where $\Pi^*(M,T)$ is the optimized expected profit of a monopoly insurer given any (M,T).[12]

2.3. Government's Decisions

The government offers a excess-of-loss per event (CAT XL) reinsurance treaty at a reinsurance premium M.[13] The reinsurance covers the catastrophe losses above the insurer's retention K ($K \geq 0$) with a coverage $\overline{K}$ ($\overline{K} \geq 0$). The upper limit of the reinsurance is thus $K + \overline{K}$. Specifically, $\overline{K} = 0$ if the government offers no reinsurance and the corresponding reinsurance premium is zero, $M = 0$. When $\overline{K} \geq l - K$, the government offers a full reinsurance that covers all losses above the insurer's retention and is essentially equivalent to unlimited reinsurance coverage.[14] The reinsurance premium M and the catastrophe tax T are endogenously determined in our model given

[12]These linear extrapolations on insurance price and expected profit capture the pricing and profitability constraints imposed by competitive pressure. In markets with imperfect competition, the insurer can raise the premium by holding more capital and earns a positive expected profit, different from a price taker. The premium $\alpha(\eta)$ is a weighted sum of individuals' maximum willingness to pay which is determined by the insurer's capital decision, and the exogenous market price $\widetilde{\alpha}$. The insurer with larger market pricing power η has a stronger impact on the premium $\alpha(\eta)$ and earns a larger share of the monopolistic expected profit. The insurer with lower market pricing power η faces a tighter profitability constraint imposed by competitive pressure. Alternative assumptions that satisfy a positive relationship between the insurance price and market pricing power $\partial\alpha(\eta)/\partial\eta > 0$ and a positive relationship between the expected profit and the market pricing power $\partial\Pi(\alpha(\eta), C, R, M)/\partial\eta > 0$ will not change the qualitative conclusions but mathematically more cumbersome.

[13]The CAT XL treaty is the type of reinsurance product most commonly used for catastrophe risks. It covers the aggregate losses caused by a single event that exceed an insurer's retention and up to an agreed coverage limit. The U.S. TRIA and French CCR programs use this type of reinsurance contract. The insurers' retention of TRIA is 20% of the direct premium income of the preceding year. The retention of CCR is 400 million euros per event.

[14]Our model allows for the specification that government offers the primary insurance by assuming all catastrophe risks and all premiums of the original insurer (i.e., $K = 0$, $\overline{K} = l$, and $M = \alpha$). Abstracting away the private insurance market, however, prevents us to analyze the optimal risk-sharing among the public, the private, and consumers. Existing literature also points out that government catastrophe insurance confronts challenges and frictions that were not present in government reinsurance, including political constraints in risk-based pricing and discriminated coverage (Zanjani, 2008), excessive operational costs for serving individual clients (Bruggeman et al., 2012), and the lack of expertise in underwriting and claim adjustment (Boyer and Nyce, 2013). Therefore, previous studies argue for the government's advantages being the reinsurer of last resort to cover the losses in excess layers (Kousky and Kunreuther, 2018). We refer to Boyer and Nyce (2013) for modeling the government-provided catastrophe insurance.

any layer of reinsurance coverage $(K, \overline{K})$, which varies from programs to programs in practice.[15]

Social welfare has been widely used as the goal of a social planner (or a central agency) in insurance markets when considering government intervention to restore or improve market functioning (see e.g., Huang and Tzeng, 2007; Einav et al., 2010; Tirole, 2012). Huang and Tzeng (2007) analyze the optimal tax deductible for net losses in insurance markets. They define the government's objective as the weighted sum of the insured's expected utility and the insurer's expected profit. Einav et al. (2010) develop a framework to analyze the efficiency of insurance markets and the welfare gains of government intervention. They define the welfare of insurance markets as the total surplus of consumers and insurer(s). Tirole (2012) uses the expected social welfare, i.e., the sum of a seller's gross utility and buyers' expected profit, as the objective function of the government. The expected social welfare of these models directly follows the standard consumer and producer theory in microeconomics.

Following Tirole (2012), we define the social welfare in a catastrophe risk market as the sum of individuals' expected utilities and the insurer's expected profit:[16]

$$V(\alpha, C, R, M, T) \equiv U_i(\alpha, T) + \Pi(\alpha, C, R, M). \tag{5}$$

Given the optimal decisions of the insurer and individuals, the optimization problem of the government is as follows:

$$\begin{aligned} &\max_{M,T} V(\alpha, C, R, M, T), \\ &s.t.\ T \geq 0,\ M \geq 0,\ \eta \Pi^*(M, T) \geq 0, \\ &\qquad T + M \geq E[I_{re}(x)]. \end{aligned} \tag{6}$$

[15]The optimal retention and limit of a reinsurance product depends on the type of catastrophe risks and on the underwriting appetite of the original insurer and the reinsurer, which vary from region to region and are adjusted from time to time. For example, the reinsurance limit of Dutch Agriver increased after the excessive agricultural damage caused by heavy rainfall in 2002 and expanded to cover frost damage to fruit farming in 2007. The insurers' retention of TRIA grew from 1% of the direct premium income per event in 2002 to 20% in 2007. Our paper focuses on the optimal government intervention policies and optimal pricing of the government reinsurance for efficient risk-sharing. We refer to Bernard (2013) for discussion on the optimal catastrophe reinsurance design.

[16]Alternatively, we consider the government's objective function as a weighted sum of individuals' expected utilities and the insurer's expected profit, i.e., $V(\alpha, C, R, M, T) \equiv \omega U_i(\alpha, T) + (1-\omega)\Pi(\alpha, C, R, M)$ (Huang and Tzeng, 2007), and the total surplus of consumers and the insurer, i.e., the sum of certainty equivalents for consumers and profits of firms (Einav et al., 2010). Our conclusions remain intact. We note that industrial organization studies usually maximize consumer surplus/utility as the goal of a social planner (Goodspeed and Haughwout, 2012; Boyer and Nyce, 2013). In our model, the specification in a perfectly competitive market allow the government to be concerned about consumers' welfare only and all our conclusions hold. The solvency regulation literature usually minimizes an insurer's default probability as the goal of a regulator. We analyze the tradeoff between social welfare and default probability in Proposition 3.

The last inequality is an actuarial fair condition (i.e., the budget and sustainability constraint) of the government reinsurance program, where the sum of catastrophe tax T and reinsurance premium M should be no less than the expected loss covered by the government reinsurance.[17] For simplicity, the model abstracts away the operational costs of government reinsurance and therefore the price of reinsurance, $T+M$, is actuarially fair.[18] The sovereign government eliminates the default risk of the reinsurance program (Froot and O'Connell, 2008; Bernard and Tian, 2009).

Given the reinsurance product, we are able to specify the insurance indemnity $I(x)$ and the reinsurance indemnity $I_{re}(x)$ below. The insurer defaults when $x > \bar{x}$ and pays its residual fund due to its limited liability. Catastrophe victims are homogeneous in loss amount and thus have equal rights to claim the residual fund (Charpentier and Le Maux, 2014).

$$I(x) \equiv \begin{cases} l, & \text{if } x \leq \bar{x}, \\ \frac{\lambda\alpha + C + R(\overline{K} - M)}{\lambda x}, & \text{if } x > \bar{x}, \end{cases} \tag{7}$$

$$I_{re}(x) \equiv \min\{\overline{K}, \max\{0, \lambda x l - K\}\}. \tag{8}$$

2.4. No Reinsurance and Private Reinsurance Cases

If no reinsurance is available in the market, our model degenerates to $T = 0$ and $\overline{K} = M = 0$. For private reinsurance, the model degenerates to $T = 0$ and thus the private reinsurance can be denoted as $(K, \overline{K}, M_p)$. To create a fair comparison with the government reinsurance, the private reinsurance premium is also actuarially fair, $M_p = E[I_{re}(x)]$, and the private reinsurer is also absent of default risk. The operational costs of private reinsurance are abstracted away for simplicity. All our conclusions hold given the same operational costs of private and government reinsurance.

In a NR equilibrium, the insurer sets the premium at α_n^e, holds the optimal capital C_n^e, and earns the expected profit Π_n^e. The catastrophe insurance market fails to exist when individuals' maximum willingness to pay $\alpha_n^*(C_n^e)$ is lower than the insurer's minimum acceptable premium $\underline{\alpha}_n(C_n^e)$. Otherwise, each individual purchases the catastrophe insurance with expected utility U_{in}^e. Therefore, the expected social utility is $V_n^e = U_{in}^e + \Pi_n^e$.

In a PR equilibrium, the insurer sets the premium at α_p^e, holds the optimal capital C_p^e, purchases the private reinsurance $R^e = 1$, and earns the expected profit Π_p^e. Individuals' maximum willing-

[17]In Section 4.2, we discuss how our government reinsurance program with a risk-based, actuarially fair, and affordable price can be sustainable and break even by itself over time.

[18]Our conclusions hold if we consider a cost loading $r' > 0$ that is not excessively large (see Section 4.2 for details).

ness to pay $\alpha_p^*(C_p^e)$ is always higher than the insurer's minimum acceptable premium $\underline{\alpha}_p(C_p^e)$ and thus each individual purchases the catastrophe insurance with expected utility U_{ip}^e. Therefore, the expected social utility is $V_p^e = U_{ip}^e + \Pi_p^e$. The details of the NR and PR equilibriums are documented in Appendices A1 and A2, respectively.

3. Equilibrium and Proposition

The equilibrium of a catastrophe risk market with government reinsurance is characterized by a set of optimal decisions made by the government, the insurer, and individuals. Given the event size $x \sim F(x|p,\delta)$, per-capita loss l, the insurer's retention K, reinsurance coverage $\overline{K}$, cost of capital r, market pricing power η, and each individual's utility function $U(\cdot)$, we derive a unique sub-game perfect equilibrium that jointly satisfies the following conditions. The proof of the equilibrium is presented in Appendix A3.

(1) The government offers the catastrophe reinsurance at the optimal balance of catastrophe tax and reinsurance premium, which satisfies its budget and sustainability constraint (i.e., the actuarial fair condition) and achieves the best balance between individuals' expected utilities and the insurer's expected profit:

$$M^e + T^e = E[I_{re}(x)], \tag{9}$$

$$-\frac{\partial}{\partial T}U_i(\alpha^e, T^e) = \frac{\partial}{\partial T}\Pi(\alpha^e, C^e, R^e, M^e). \tag{10}$$

Figure 2 illustrates the government's optimal balances between catastrophe tax and reinsurance premiums to maximize the expected social utility:

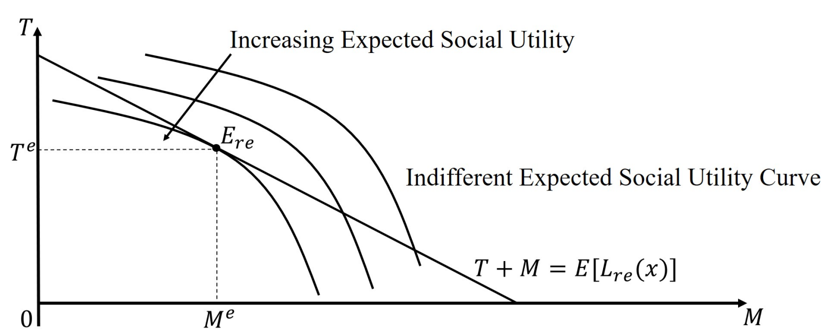


Figure 2: Tradeoff between Catastrophe Tax and Reinsurance Premium

Note: The curves represent indifferent expected social utilities. The line represents the actuarially fair condition in Eq.(9). Both catastrophe tax and reinsurance premium decrease the expected social utility and thus the curve is concave to the origin. The tangent point E_{re} is the equilibrium in a catastrophe reinsurance market.

(2) Given the optimal catastrophe tax and reinsurance premium, the insurer purchases the government reinsurance:

$$R^e = 1. \tag{11}$$

The equilibrium conditions to define the insurer's optimal decisions on the insurance premium α^e and the holding capital C^e differ in the market structure, that is captured by its market pricing power η.

(i) In a monopolistic market where $\eta = 1$, the insurer sets the catastrophe insurance premium at the maximum willingness to pay of each individual and holds the optimal capital such that its marginal benefit is equal to its marginal cost:

$$\alpha^* = \alpha^*(C^*, T^*), \tag{12}$$

$$F(\bar{x}^*)\frac{\partial}{\partial C}\alpha^*(C^*, T^*) = 1 - F(\bar{x}^*) + r, \tag{13}$$

where $\bar{x}^* = \frac{1}{l}(\alpha^* + C^* + \overline{K} - M^*)$, $\alpha^* \equiv \alpha^e(\eta = 1)$, $C^* \equiv C^e(\eta = 1)$, $T^* \equiv T^e(\eta = 1)$, and $M^* \equiv M^e(\eta = 1)$. In Eq.(13), $F(\bar{x}^*)\frac{\partial}{\partial C}\alpha^*(C^*, T^*)$ is the marginal increase in individuals' maximum willingness to pay (or equivalently, the marginal increase in the monopolistic insurer's premium income) with respect to capital. The right-hand side $1 - F(\bar{x}^*) + r$ is the marginal cost of capital (i.e., the marginal expected cost of losing all capital due to default plus the marginal cost of carrying capital). The insurer's expected profit is maximized when the marginal benefit of capital is equal to its marginal cost.

(ii) In a perfectly competitive market where $\eta = 0$, the insurer, as a price taker, sells the catastrophe insurance at the market price $\alpha^e(\eta = 0) = \widetilde{\alpha}$ and determines its optimal capital such that its expected profit is zero:

$$\int_0^{\bar{x}^{**}} [\widetilde{\alpha} - M^{**} - xl + I_{re}(x)] f(x)dx - [1 - F(\bar{x}^{**})]C^{**} - rC^{**} = 0, \tag{14}$$

where $\bar{x}^{**} = \frac{1}{l}(\widetilde{\alpha} + C^{**} + \overline{K} - M^{**})$, $C^{**} \equiv C^e(\eta = 0)$, and $M^{**} \equiv M^e(\eta = 0)$.

(iii) In markets with imperfect competition where $0 < \eta < 1$, the insurer sets the catastrophe insurance premium $\alpha^e(\eta)$ at the weighted sum of individuals' maximum willingness to pay and the exogenous market price and holds the optimal capital $C^e(\eta) = C^e_{premium}(\eta) + C^e_{quality}(\eta)$ such that:

$$\alpha^e(\eta) = \eta\alpha^*(C^e(\eta), T^e(\eta)) + (1 - \eta)\widetilde{\alpha}, \tag{15}$$

$$\eta F(\hat{x}(\eta))\frac{\partial}{\partial C}\alpha^*\left(C^e_{premium}(\eta), T^e(\eta)\right) = 1 - F(\hat{x}(\eta)) + r, \tag{16}$$

$$\int_0^{\bar{x}^e(\eta)} [\alpha^e(\eta) - M^e(\eta) - xl + I_{re}(x)] f(x) dx - [1 - F(\bar{x}^e(\eta))][C^e_{premium}(\eta) + C^e_{quality}(\eta)] \\ - r[C^e_{premium}(\eta) + C^e_{quality}(\eta)] = \eta \Pi^* (M^e(\eta), T^e(\eta)), \tag{17}$$

where $\hat{x}(\eta) = \frac{1}{l}[\alpha^*(C^e_{premium}(\eta), T^e(\eta)) + C^e_{premium}(\eta) + \overline{K} - M^e(\eta)]$ and $\bar{x}^e(\eta) = \frac{1}{l}[\alpha^e(\eta) + C^e(\eta) + \overline{K} - M^e(\eta)]$. The optimal capital $C^e(\eta)$ consists of two components: (i) the premium income capital $C^e_{premium}(\eta)$ and (ii) the product quality capital $C^e_{product}(\eta)$. As shown in Eq.(16), the insurer holds the optimal premium income capital $C^e_{premium}(\eta)$ such that the marginal increase in its premium income equals to the marginal cost of capital, which optimizes its expected profit without considering any competitive pressure on profit. As shown in Eq.(17), the insurer holds the optimal product quality capital $C^e_{quality}(\eta)$ in addition to $C^e_{premium}(\eta)$ such that its expected profit (i.e., the left hand side of Eq.(17)) meets the profitability constraint.

The insurer's optimal decisions with different market structures (i.e., in different market pricing power η) are smoothed and inherently consistent. The product quality capital decreases to 0 without competitive pressure in a monopolistic market (i.e., $C^e_{quality}(\eta = 1) = 0$ and $C^* = C^e(\eta = 1) = C^e_{premium}(\eta = 1)$) because the insurer has no incentive to pass the benefits of government reinsurance to individuals but earns a maximized expected profit $\Pi^*(M^*, T^*)$. The premium income capital decreases to 0 with perfect competition and zero expected profit (i.e., $C^e_{premium}(\eta = 0) = 0$ and $C^{**} = C^e(\eta = 0) = C^e_{quality}(\eta = 0)$) because holding more capital cannot increase premium income under competitive pressure (i.e., $\eta \frac{\partial}{\partial C} \alpha^* \left(C^e_{premium}(\eta), T^e \right) = 0$ when $\eta = 0$). The insurer has to compete for customers by product quality and thus holds maximal product quality capital.

(3) Given the optimal decisions of the government and the insurer, each individual purchases the catastrophe insurance:

$$\lambda^e = 1. \tag{18}$$

It is optimal for the insurer to insure all individuals because (i) all individuals are homogeneous, (ii) the per-policy expected profit is independent of the size of the insurance portfolio, and (iii) the insurer earns a non-negative per-policy expected profit from any individual insured. Therefore, the insurer would be willing to insure as many individuals as possible.

Given the above optimal decisions in the equilibrium, each individual ends up with a maximized expected utility U^e_i, the insurer earns an optimal expected profit Π^e, and thus the maximized expected social utility is $V^e = U_i^e + \Pi^e$. In Appendix A4, we consider a more general expected social utility function that weighted sums U_i^e and Π^e. All our propositions hold.

Figure 3 illustrates an individual's expected utility and the insurer's expected profit in equilibriums under different market structures. In a perfectly competitive market (i.e., $\eta = 0$), the insurer earns zero expected profit and each individual achieves the highest expected utility. As the market becomes less competitive, the insurer with some market pricing power (i.e., $0 < \eta < 1$) earns a positive expected profit and each individual ends up with a lower expected utility. In a monopolistic market (i.e., $\eta = 1$), the insurer earns the highest expected profit and each individual ends up with the uninsured expected utility.

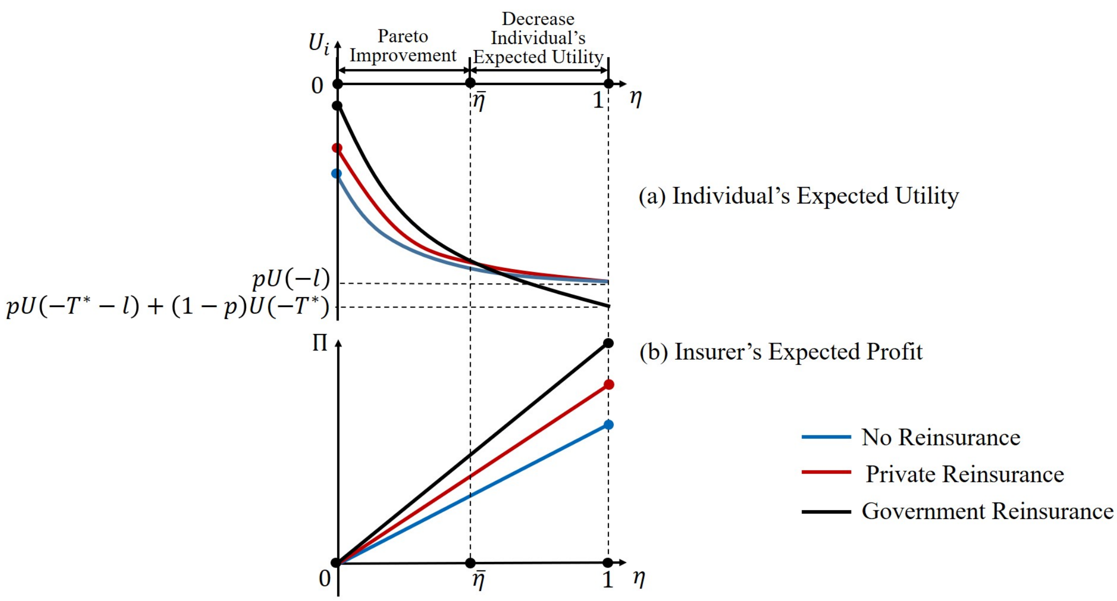


Figure 3: Government Reinsurance in Different Market Structures

Independent of the market structure, the government reinsurance $(K, \overline{K}, M^e, T^e)$ reallocates the catastrophe risk between the insurer and the government. For any reinsurance product $(K, \overline{K})$ that satisfies the condition $K + \overline{K} \leq \bar{x}l$,[19] there exists a unique pair of M^e and T^e in the equilibrium. This result suggests that the government reinsurance is applicable to a wide range of catastrophe risk layers and therefore can be applied on various types of catastrophe risks with different distributions.

We derive the following three propositions by comparing the equilibrium of government reinsurance with the NR and PR equilibriums. Proofs of propositions are presented in Appendix B.

[19]The condition implies that the insurer is always able to pay for the losses within its own retention, and its default only occurs when the insured losses of a catastrophe event exceeds the upper limit of the reinsurance.

Proposition 1. ***Existence of catastrophe insurance market***

Both private and government reinsurance ensures that the insurer is always willing to enter the catastrophe insurance market, while the market may not exist when there is no reinsurance:

$$\begin{cases} \alpha_n^*(C_n^e) < \underline{\alpha}_n(C_n^e), & \text{iff } \delta > \underline{\delta} \text{ and } r > \underline{r}, \\ \alpha_n^*(C_n^e) \geq \underline{\alpha}_n(C_n^e), & \text{if otherwise,} \end{cases} \tag{19}$$

$$\alpha_p^*(C_p^e) > \underline{\alpha}_p(C_p^e), \forall \delta \text{ and } \forall\, r, \tag{20}$$

$$\alpha^*(C^e, T^e) > \underline{\alpha}(C^e, M^e), \forall \delta \text{ and } \forall\, r. \tag{21}$$

The private catastrophe insurance market alone fails to exist, i.e., individuals' maximum willingness to pay is smaller than the insurer's minimum acceptable premium, when the following two conditions are jointly satisfied: (i) the risk correlation exceeds a certain threshold $\underline{\delta}$ ($\underline{\delta} > 0$) and (ii) the cost of capital exceeds a certain threshold $\underline{r}$ ($\underline{r} > 0$). A high correlation among individual risks results in a high default probability for the insurer to lose all its capital, and thus a low maximum willingness to pay and a higher minimum acceptable premium. A high cost of capital results in a high minimum acceptable premium (recall that the minimum acceptable premium covers the sum of expected loss and cost of capital). Proposition 1 suggests that the failure of the private catastrophe insurance market, or more broadly the under-insurance of catastrophe risks, is a joint result of the high risk correlation and the expensive capital cost. Existing literature explains the failure in catastrophe insurance market with either risk correlation (Ibragimov et al., 2009; Kousky and Cooke, 2012) or capital cost (Jaffee and Russell, 1997; Zanjani,2002), however, either reason alone will not fail the catastrophe insurance market.[20]

Reinsurance, whether private or provided by the government, closes the gap between an individual's maximum willingness to pay and the insurer's minimum acceptable premium, and thus ensures the existence of the private catastrophe insurance market. Reinsurance acts the same reducing-default-probability function as the holding capital for the catastrophe insurer but with lower cost. When the insurer has some market pricing power (i.e., $0 < \eta \leq 1$), reinsurance substitutes for holding capital, saves the capital cost, and thus lowers the minimum acceptable premium. When the insurer faces some competitive pressure (i.e., $0 \leq \eta < 1$), reinsurance increases individuals' maximum willingness to pay by adding safety buffers on top of the insurer's capital.

[20] In a multi-period setup, the inter-temporal correlation of catastrophe losses may result in the failure of private insurance market even with reinsurance (see Section 5.2 for details).

Proposition 2. ***Improvement in expected social utility***

Government reinsurance improves the expected social utility compared to the NR and PR cases:

$$V_n^e < V_p^e < V^e. \tag{22}$$

Government reinsurance achieves Pareto improvement comparing to the NR and PR cases in some more competitive catastrophe insurance markets; government reinsurance reduces individuals' expected utility in some less competitive markets:

$$\begin{cases} U_{in}^e < U_{ip}^e \leq U_i^e \text{ and } \Pi_n^e \leq \Pi_p^e \leq \Pi^e, & \text{when } 0 \leq \eta \leq \overline{\eta}, \\ U_{in}^e \leq U_{ip}^e > U_i^e \text{ and } \Pi_n^e < \Pi_p^e < \Pi^e, & \text{when } \overline{\eta} < \eta \leq 1. \end{cases} \tag{23}$$

As shown in Proposition 2 and Figure 3, private reinsurance achieves Pareto improvement compared to no reinsurance. In a monopolistic market, reinsurance acts a one-to-one substitute for holding capital, and therefore saves the insurer's cost of capital and improves its expected profit. As the market becomes more and more competitive, the competitive pressure limits the insurer's profitability. The insurer cannot use reinsurance to only substitute its holding capital but also to add some safety buffer on top of its holding capital, which improves both the insurer's expected profit and individuals' expected utilities. In a perfectly competitive market, the insurer expected profit is always zero, and reinsurance cannot replace capital but adds safety buffer, improves the product quality, and improves individuals' expected utilities.

Next, we compare the government reinsurance with the private reinsurance. By balancing between catastrophe tax and reinsurance premium, the government achieves the optimal balance between individuals' expected utilities and the insurer's expected profit, and therefore further improve the expected social utility $V_p^e < V^e$. However, this improvement is at the expense of a catastrophe tax that reduces individuals' expected utilities when competition is insufficient. In practice, a catastrophe risk market often features low competition (Emons, 2001; Zanjani, 2002). As shown in Figure 3, both individuals and the insurer can improve their expected utilities and expected profit with government reinsurance in some more competitive markets where $0 \leq \eta \leq \overline{\eta}$, thus achieving Pareto improvement. The improvement in expected social utility comes from two channels: the product quality-individual utility channel and the capital cost-insurer profit channel, which are analyzed in detail in Section 4.1.

Proposition 3. ***The insurer's default probability***

Government reinsurance decreases the insurer's default probability compared to the NR and PR *cases in some more competitive markets; government reinsurance may increase or decrease the insurer's default probability in some less competitive markets:*

$$\begin{cases} \overline{x}_n^e < \overline{x}_p^e < \overline{x}^e, & \text{when } 0 \leq \eta < \widetilde{\eta}, \\ \overline{x}_n^e \leq \overline{x}_p^e \lesseqgtr \overline{x}^e, & \text{when } \widetilde{\eta} \leq \eta \leq 1. \end{cases} \tag{24}$$

Private reinsurance decreases the insurer's default probability (i.e., increases the insurer's default threshold) compared to no reinsurance. As shown in Eq.(4), the default threshold is determined by the sum of the insurer's premium income, holding capital, and the net coverage of reinsurance. In a monopolistic market where $\eta = 1$, reinsurance acts as a one-to-one substitute for the insurer's capital and thus changes neither the sum of the holding capital and net reinsurance coverage nor individuals' maximum willingness to pay (i.e., the monopolistic insurer's premium income). Thus, its default threshold is the same as that in the NR case. In a competitive market where $0 \leq \eta < 1$, reinsurance not only substitutes the insurer's holding capital but also adds some safety buffer on top of the capital, which increases the sum of the holding capital and net reinsurance coverage, and increases individuals' maximum willingness to pay that partially becomes the insurer's premium income in markets with imperfect competition.

Compared to private reinsurance, government reinsurance further increases the insurer's product quality capital $C_{quality}$ and thus decreases its default probability through the product quality channel. However, government reinsurance also decreases the insurer's premium income capital $C_{premium}$ and thus increases its default probability through the capital cost channel. We discuss the details of these two channels in Section 4.1. There is also some wealth transfer effect that the catastrophe tax increases individuals' maximum willing to pay and, hence, the insurer's premium income when the insurer has some pricing power ($0 < \eta \leq 1$). The increased premium income further decreases the insurer's default probability.

In some more competitive markets where $0 \leq \eta < \widetilde{\eta}$, the capital cost channel is weak and thus the government reinsurance decreases the insurer's default probability, driven by the aggregate impact of the product quality channel and the wealth transfer effect. In some less competitive markets where $\widetilde{\eta} \leq \eta \leq 1$, the product quality channel is weak and the impact of government reinsurance on the insurer's default probability is driven by the opposite effect of the capital cost channel and the wealth transfer effect. Their aggregate impact is thus undetermined, depending on the degree of individuals' risk aversion. With increasing absolute risk aversion (IARA), individuals have lower insurance demand for large-loss events (Zhou et al., 2010), i.e., lower maximum willingness to pay and thus lower premium income for the insurer. Thus, the wealth transfer effect is weak, the capital cost channel dominates the increase in the insurer's default probability. On the contrary, when individuals exhibit decreasing or constant absolute risk aversion (DARA or CARA), the impact of the wealth transfer effect (and the weakened product quality channel) dominates the capital cost channel and thus government reinsurance decreases the insurer's default probability.

In practice, a catastrophe risk market often features low competition (Emons, 2001; Zanjani, 2002) and IARA individuals (Gollier, 1997). Individuals are used to underestimate catastrophe risks (Froot, 2001; Kunreuther, 2015) and thus are less risk averse for large-loss events, implying an IARA preference (Zhou et al., 2010). Thus, in a catastrophe risk market, it is likely that the government reinsurance introduces a tradeoff between an improved expected social utility, and (i) an increase in the insurer's default probability and (ii) an decrease in individuals' expected utilities (see Proposition 2). To avoid this dilemma, competition in the private catastrophe insurance market should be encouraged, which, according to Propositions 2 and 3, mitigates the potential problems of the government reinsurance.

4. Impact Channels and Pricing of Government Reinsurance

4.1. Impact Channels

Figure 4 shows that the government reinsurance and the tradeoff between catastrophe tax and reinsurance premium improve expected social utility through the product quality and the capital cost channels, compared to private reinsurance. The impact of these two channels is, however, opposite on the insurer's default probability. This pair of channels works on different components of the insurer's holding capital: product quality capital $C_{quality}$ and premium income capital $C_{premium}$.

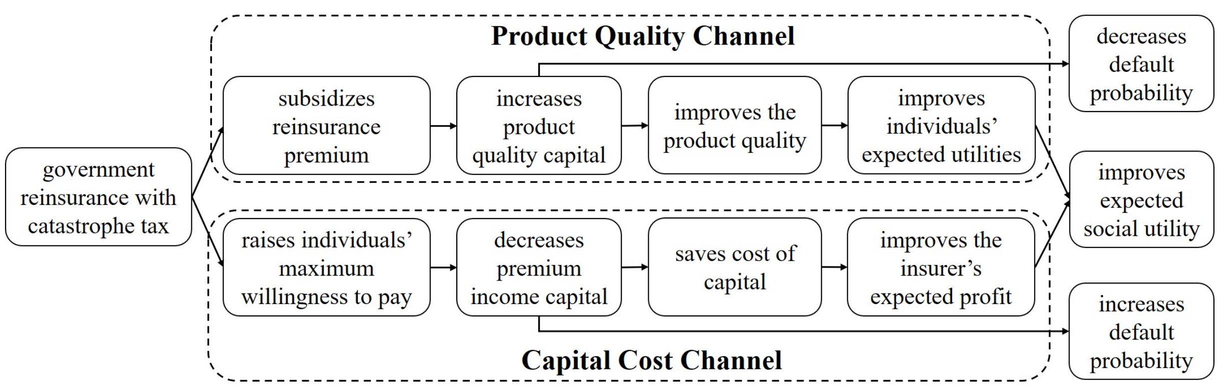


Figure 4: Impact Channels of Government Reinsurance

The strengths of the two channels vary, depending on the market structure. The product quality channel is stronger when the market is more competitive, while the capital cost channel is stronger when the market is close to monopoly. In markets with imperfect competition, both channels coexist. In a monopoly market where the insurer is a price maker, the product quality channel shuts down as $C^e_{quality,p}(\eta = 0) = C^e_{quality}(\eta = 0) = 0$; in a perfectly competitive market where the insurer is a price taker, the capital cost channel shuts down as $C^e_{premium,p}(\eta = 1) = C^e_{premium}(\eta = 1) = 0$.[21] We formalize the two channels and present their intuitions as follows.[22] The proofs of the two channels are provided in Appendices C1 and C2.

Channel 1. ***Product Quality Channel***

Government reinsurance raises the insurer's product quality capital compared to private reinsurance, and strictly so when the catastrophe insurance market is not monopolistic:

$$\begin{cases} C^e_{quality,p}(\eta) < C^e_{quality}(\eta), & \text{when } 0 \leq \eta < 1, \\ C^e_{quality,p}(\eta) = C^e_{quality}(\eta) = 0, & \text{when } \eta = 1. \end{cases} \tag{25}$$

The product quality channel works on the insurer's product quality capital to improve the expected social utility and to decrease the insurer's default probability. The catastrophe tax subsidizes the insurer's reinsurance premium. The saved premium of reinsurance can be used to support the insurer to hold more product quality capital. Additional product quality capital reduces the insurer's default probability and delivers a safer (higher quality) product to individuals, which increases individuals' expected utilities and, hence, the social welfare. As the market becomes more and more competitive, the insurer passes a larger fraction of its benefits from the catastrophe tax subsidy to individuals by holding more product quality capital. In the two extreme market structures, monopoly imposes no pressure on the insurer to share its benefits (i.e., the product quality channel shuts down); perfect competition forces the insurer to pass all its benefits to individuals (i.e., a maximum capital cost channel).

Channel 2. ***Capital Cost Channel***

Government reinsurance reduces the insurer's premium income capital compared to private rein-

[21] $C^e_{quality,p}(\eta)$ and $C^e_{premium,p}(\eta)$ are the insurer's product quality capital and premium income capital in the PR equilibrium, respectively.

[22] Although the wealth transfer impact of tax on expected social utility is undetermined, depending on individuals' initial wealth and on their degree of risk averse, it is always dominated by the product quality and premium income channels and thus cannot change the impact direction on expected social utility.

surance, and strictly so when the catastrophe insurance market is not perfectly competitive:

$$\begin{cases} C^e_{premium,p}(\eta) > C^e_{premium}(\eta), & \text{when } 0 < \eta \leq 1, \\ C^e_{premium,p}(\eta) = C^e_{premium}(\eta) = 0, & \text{when } \eta = 0. \end{cases} \tag{26}$$

The capital cost channel works on the insurer's premium income capital to improve the social welfare and to increase the insurer's default probability. The catastrophe tax increases individuals' maximum willingness to pay, (part of) which becomes the insurer's premium income when the insurer has some market pricing power, and thus offsets the insurer's incentive to hold more premium income capital. The decreased premium income capital increases the insurer's default probability but saves the insurer's cost of capital, increases the insurer's expected profit and, hence, the social welfare. As the market becomes less and less competitive, the insurer has more market pricing power (i.e., more like a price maker than a price taker) and thus is more capable of exploiting individuals' maximum willingness to pay and holds less premium income capital. In the two extreme market structures, perfect competition does not allow the insurer to price or to share any increments in individuals' maximum willingness to pay (i.e., the capital cost channel shuts down) and monopoly enables the insurer to take all the increments (i.e., a maximum capital cost channel).

4.2. Pricing of Government Reinsurance

In this section, we show that the pricing of government reinsurance is risk based, affordable, and long-term sustainable. Proofs of properties of pricing of government reinsurance are presented in Appendix C3. The government reinsurance price comprises two components: the catastrophe tax T and the reinsurance premium M. As shown in Eq.(9), the reinsurance price is risk-based and equals to the expected reinsurance loss given a set of exogenous reinsurance coverage $(K, \overline{K})$. The price is higher with a lower retention K, a larger coverage $\overline{K}$, and/or a greater potential loss l.

$$\frac{\partial(M^e + T^e)}{\partial K} < 0, \frac{\partial(M^e + T^e)}{\partial \overline{K}} > 0, \frac{\partial(M^e + T^e)}{\partial l} > 0. \tag{27}$$

The government reinsurance is affordable for the private catastrophe insurer as the reinsurance premium is always lower than the primary insurance premium, i.e., $M^e < \alpha^e$. This is because (i) the reinsurance premium is less than the expected reinsurance loss given the catastrophe tax subsidy; (ii) the primary premium is higher than the expected loss of catastrophe insurance as individuals are risk averse and willing to pay more than fair price for the catastrophe coverage; (iii) the reinsurance covers a part of the risk under the primary insurance.

To analyze the sustainability of the government reinsurance program, we repeat the single-period game over time: In each period, a share of individuals x_t (x_t is independent across periods) suffers a loss l with probability p. Individuals, the insurer, and the government repeat the same decision process as shown in Figure 1. The government's budget and sustainability constraint (i.e., the actuarial fair condition) remains binding in each period based on the expected loss. Therefore, the government reinsurance is sustainable and break-even in the long run, given a large number of individuals and according to the Law of Large Numbers. The government reinsurance program accumulates reserves from catastrophe taxes and reinsurance premiums before a catastrophe event occurs and covers the catastrophe losses using the reserves and premiums once the catastrophe hits. When the accumulated reserves cannot cover the loss in full, a private reinsurer may thus be at the risk of bankruptcy, whereas a sovereign government is able to inter-temporally smooth the volatilities with its non-bankruptcy nature by borrowing from other public or private sources. The government reinsurance program can repay the loans with catastrophe taxes and reinsurance premiums in subsequent periods. The government reinsurance program is thus self-sustainable.

The government reinsurance program remains sustainable if certain costs are loaded on the actuarially fair reinsurance premium, i.e., $T' + M' = (1+r')E[I_{re}(x)]$ ($r' > 0$). All our propositions and channels hold as long as (i) the rate of cost loadings is not excessively high such that the cost of government reinsurance is less than the capital cost saved by the reinsurance, i.e., $r'E[I_{re}(x)] < r(\overline{K} - (M' + T'))$ and the rate of cost loading is less than the capital cost threshold of the market failure, i.e., $r' < \underline{r}$, and (ii) private reinsurance has the same rate of cost loadings, i.e., $M'_p = (1+r')E[I_{re}(x)]$.

The cost loading extension can also be used to analyze the returns of reserve investments and the interest rate for inter-temporal loans of the government reinsurance program. When the investment return rate is lower than (equal to) the loan interest rate, it is equivalent to $r' > 0$ ($r' = 0$) and our conclusions hold with the above conditions. When the investment return rate is higher than the loan interest rate, the government reinsurance earns a surplus and thus can offer reinsurance with a lower price, which further improves the social welfare.

5. Application to the COVID-19 Pandemic

In this section, we extend and apply the model to analyze the risks of COVID-19 and future pandemics, and their corresponding (re)insurance solutions. We connect our catastrophe risk model with the classical SIR model in the public health literature and calibrate it in the context of COVID-19. We extend the model to capture the inter-temporal correlation of of the losses from COVID-19 and other pandemics, which is one of the key differences between pandemics and many natural catastrophes. Last but not least, we compare two government backed (re)insurance solutions for pandemic-caused business interruptions: the Pandemic Risk Insurance Act (PRIA) versus the Business Continuity Protection Program (BCPP).

5.1. Calibration with the COVID-19 Pandemic

The COVID-19 and other pandemics are a type of catastrophe risk because most individuals in modern society are exposed to the risk of infection and the individual risks are highly correlated due to the communicable nature of an epidemic. As a result, the aggregate losses of the COVID-19 exceed those of many natural catastrophes, in particular due to large medical expenses and mass business interruptions, which challenge the financial flexibility and solvency of insurers and reinsurers (Swiss Re, 2020).

To analyze the performance of government reinsurance in a COVID-19 type of pandemic risk environment, we connect our catastrophe risk model with the Susceptible-Infected-Recovered (SIR) model that is widely used in the epidemiology and public health literature. The basic reproduction number R_0 in an SIR model captures the number of secondary cases, which one case would produce in a completely susceptible population. R_0 is widely used to capture the severity of a communicable disease (see e.g., Gollier, 2020; Pindyck, 2020). The relationship between the basic reproduction number R_0 and the cumulative infection rate p_p can be expressed as follows (Kermack and Mekendrick, 1927; Pindyck, 2020):

$$-ln(1-p_p) = R_0 p_p. \tag{28}$$

Following Charpentier and Le Maux (2014), the correlation of individual risks δ in our catastrophe risk model can be further specified as $\delta = 1 - \frac{p_N}{p_C}$, where p_N is the probability that an individual claims a loss without a catastrophe/pandemic and p_C is the probability that an individual claims a loss given that a catastrophe/pandemic occurs. In the context of pandemic risks and health insurance markets, additional health insurance claims caused by a pandemic can be captured by the cumulative infection rate p_p. Thus, the probability that an individual claims a loss given a pandemic is the sum of the baseline claim rate and the cumulative infection rate of the pandemic, i.e., $p_C = p_N + p_p$.[23] Therefore, the relationship between the correlation of individual risks δ and the cumulative infection rate of a pandemic p_p can be derived as follows. It is easy to see that p_p and δ are positively correlated and $\delta \to 0$ as $p_p \to 0$.

$$\delta = 1 - \frac{p_N}{p_N + p_p}. \tag{29}$$

We thus derive the relationship between the basic reproduction number R_0 in a SIR model and the risk correlation δ in our catastrophe risk model as follows. We see that R_0 and δ are positively correlated and $\delta \to 0$ as $R_0 \to 1$.

$$-ln\left(1 - \frac{p_N}{1-\delta} + p_N\right) = R_0\left(\frac{p_N}{1-\delta} - p_N\right). \tag{30}$$

Next, we parameterize the COVID-19 pandemic by R_0 and p_p to calibrate the model and to quantify the impact of COVID-19 and its (re)insurance solutions on social welfare. The model parameters based on various COVID-19 scenarios are presented in Table 1. The loss l of each individual is normalized to 1. Therefore, the maximum losses to the unit mass of individuals follows $l = 1$ as we normalize the population to 1. We adopt Charpentier and Le Maux's (2014) risk distribution function $F(x) = \sum_{j=0}^{\lfloor 1000x \rfloor} \binom{1000}{j} \left[0.9 p_N^j (1-p_N)^{1000-j} + 0.1 p_C^j (1-p_C)^{1000-j}\right]$[24] and their exponential utility function $U(Y) = 1 - e^{(-0.6Y)}$ for each individual exposed to the pandemic risk. We consider a monopolistic pandemic insurance market where $\eta = 1$ because pandemic risks "are too widespread, too severe, and too unpredictable for the insurance industry to underwrite" (APCIA et al., 2020) and few (re)insurers can offer private pandemic insurance (Munich Re, 2020). Alternatively, we report the results in Appendix D1 with η=0.75, 0.5, and 0.25, which are consistent with our main results. The cumulative infection rate p_p of COVID-19 ranges in (0%,70%) based on the reported statistics in different regions as of November 15, 2020 and the estimated results in the literature.

[23] We assume that (i) all COVID-19 infected individuals with health insurance will claim a loss, (ii) the distribution of the cumulative infection rate is independent of health coverage, and (iii) there is no overlap between individuals claiming a loss without a pandemic and individuals infected by the pandemic. Our conclusions remain intact if the above assumptions are violated, but this would cause the simulation process to become cumbersome.

[24] In the context of a pandemic-catastrophe risk, x is the share of the population claiming a loss, which includes the COVID-19 infected individuals and individuals claiming for other diseases.

Table 1: Calibrated Parameters

Parameter	Definition/Location	Value	Source
η	The insurer's market pricing power (inverse measure of market competition)	1	Charpentier and Le Maux (2014)
r	Unit cost of capital	0.05	Zanjani (2002)
K	The insurer's retention of CAT XL reinsurance	0.1	10% of the loss l
$\overline{K}$	Coverage of CAT XL reinsurance	0.2	20% of the loss l
p_N	Probability that an individual claims a loss without a pandemic (baseline claim rate)	10%	Baseline claim rate of health insurance with a high deductible
p_p	Worldwide (Nov. 15, 2020)	0.7%	WHO (2020)
	Germany (Nov. 15, 2020)	0.9%	WHO (2020)
	Italy (Nov. 15, 2020)	1.9%	WHO (2020)
	U.S. (Nov. 15, 2020)	3.2%	WHO (2020)
	Andorra (Nov. 15, 2020)	7.4%	WHO (2020)
	North Dakota (Nov. 15, 2020)	8.5%	CDC (2020)
	New York (estimated)	26%	Fernández-Villaverde and Jone (2020)
	Worldwide (Spanish Flu)	33%	Taubenberger and Morens (2006)
	Worldwide (estimated)	40%-70%	CBS (2020)
	Italy (estimated)	47.1%	Manski and Molonari (2020)
	Illinois (estimated)	52.2%	Manski and Molonari (2020)
	U.S. (estimated)	58%	Pindyck (2020)

Figures 5(a) through 5(c) show that the expected social utility decreases as the basic reproduction number, the cumulative infection rate, and the individual risk correlation increase. In other words, social welfare decreases as the COVID-19 pandemic spreads. Government reinsurance improves the expected social utility compared to the cases with no reinsurance and private reinsurance, and this improvement becomes more prominent as the pandemic spreads. However, the efficiency gains from government reinsurance are insufficient to compensate the welfare losses when the pandemic becomes massively widespread. In particular, as shown in Figure 5(c), the expected social utility drops sharply when the individual risk correlation δ exceeds 0.7 or, equivalently, when the cumulative infection rate p_p exceeds 25%. Compared to the Spanish Flu with $p_p = 33\%$ (Taubenberger and Morens, 2006), the COVID-19 is estimated to be a greater pandemic, infecting 40%-70% of the world population (CBS, 2020). The results strongly suggest that any pandemic risk-sharing solutions, including government or private reinsurance, should be coupled with social distancing and other public health measures to control the spread of the pandemic.

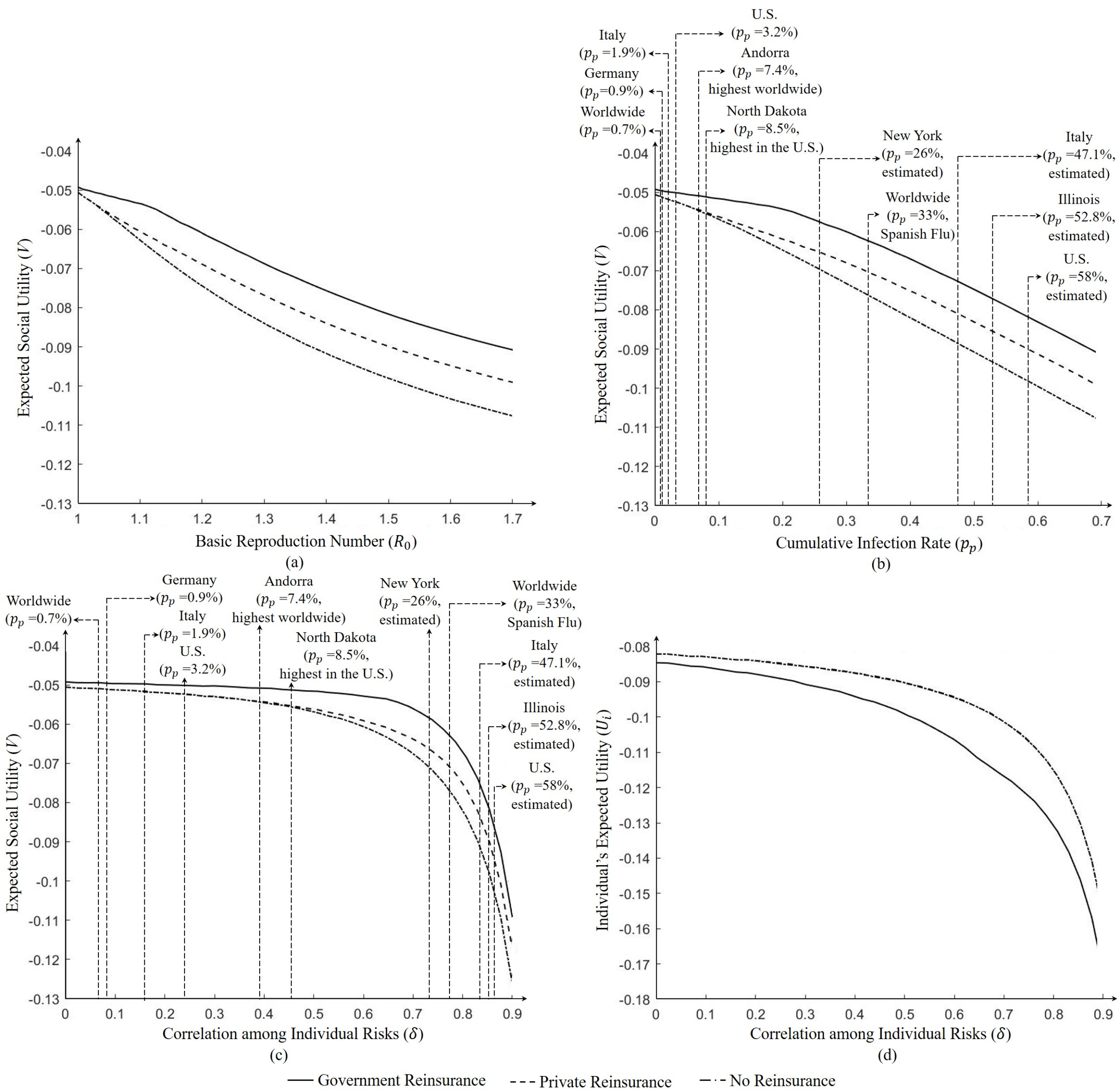


Figure 5: COVID-19, Expected Social Utility, and Individual's Expected Utility

Note:In Figure 5(a), the basic reproduction number R_0 is assumed to be constant in our calibration and its range matches the range of the cumulative infection rate p_p. The R_0 might be smaller than some estimated R_0 based on certain pandemic-outbreak time periods and policy circumstances (e.g., R_0=1.6, 1.8, 2.0, 2.2, 2.5, 2.8, and 3.0 in Atkeson, 2020). We refer to Gollier (2020) for analyses on policy-sensitive and time-variant basic reproduction numbers. In Figure 5(d), curves for no reinsurance and private reinsurance coincide because individuals end up with the expected utility of not buying pandemic insurance $pU(-l)$ in a monopolistic market in both cases.

Figure 5(d) shows that individuals' expected utilities decreases as the correlation among individual risks increases in a monopolistic insurance market. Government reinsurance further decreases individuals' expected utilities compared to the cases with no reinsurance and private reinsurance. In other words, the improvement in social welfare with government reinsurance comes with the expense in individuals' expected utilities. Therefore, anti-monopoly and/or price regulation policies should always be coupled with the government pandemic reinsurance to balance the interests of individuals and insurers and to achieve a Pareto improvement.

5.2. Inter-temporal Correlation of Pandemic Losses

As we have learned from the COVID-19 pandemic, one of the major differences between a pandemic and many natural catastrophes is that the pandemic losses are correlated inter-temporally. The losses caused by a hurricane in one period are usually independent of the hurricane losses in the next period. However, the losses caused by a pandemic in one period are positively correlated with the pandemic losses in the following period because a pandemic spreads more quickly and widely as the number of infected individuals increases. In other words, a pandemic persists and becomes harder to eliminate once it breaks out.

To capture the inter-temporal correlation of pandemic losses, we extend the model to two periods in the context of business interruption insurance. In each period, a unit mass of businesses, the private insurer, and the government repeat the decision process in Figure 1. Each business also decides whether to open or close in addition to its insurance purchasing decision. The potential losses of business interruption due to a pandemic are realized after all decisions are made.

Business interruption insurance covers the loss of profits triggered by an insured event. A standard business interruption policy usually excludes communicable diseases, but these can be covered via an extension with an additional premium (Hartwig et al., 2020). Private reinsurers have developed risk transfer solutions for epidemic-caused business interruptions, delays in start-up, and temporary site closures in order to support private insurers' ability to offer communicable disease coverage. "The idea is fairly simple—and actually not too far removed from our expertise in risk managing natural disasters" (Munich Re, 2020). The communicable disease extension covers business interruption losses due to an epidemic/pandemic but does not cover business closings that occur for a purely precautionary purpose because physical losses are required to trigger the indemnity.[25] Our two-period model incorporates the above business interruption policy, including its communicable disease extension and refers to it as "pandemic insurance".

Recall that in the single-period game, the losses to the unit mass of businesses are xl where $x \sim F(x|p,\delta)$. The expected losses of the businesses are $E(x)l = pl$. Figure 6(a) models a catastrophe

[25] Since the COVID-19 breakout, eight state courts in the U.S. have concluded that COVID-related business losses constitute physical loss and thus should be covered by business interruption insurance despite virus exclusions (Reuters, 2020).

risk without considering the inter-temporal correlation of catastrophe losses: the expected loss in each period is pl when the businesses decide to open and l when they close proactively. The expected losses are thus independent of catastrophe events in the previous period. Figure 6(b) allows the expected loss in a period to be dependent on catastrophe events in the previous period. Thus, given a catastrophe event in the previous period, the expected loss of businesses in period 1 is ql $(p \leq q < 1)$ when they decide to open and l when they close proactively.

The two-period setup allows us to introduce the tradeoff between economic development (open business) and the spread of the pandemic. Opening businesses during a pandemic will cause the pandemic to continue in the next period while closing them will stop it. Therefore, the expected losses in period 2 are $[q^2 + (1-q)p]l$ when the businesses decide to remain open in period 1 and period 2; pl when the businesses decide to close in period 1 and open in period 2 (because the pandemic came under control due to business closure in period 1); l when the businesses close proactively in period 2. Thus, $q - p$ captures the inter-temporal correlation of pandemic losses, i.e., a larger $q - p$ implies higher inter-temporal correlation and $q = p$ implies no inter-temporal correlation.

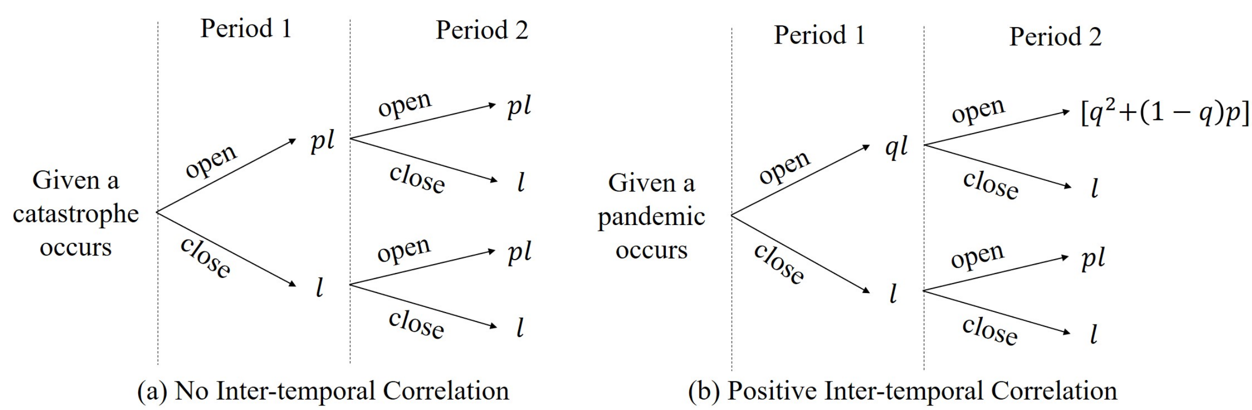


Figure 6: Two-period Model

We derive the equilibrium and policy implications of the extended two-period model in the following four corollaries. The proofs are provided in Appendix D2.

Corollary 1.1. ***The equilibrium***

In period 1, businesses choose to remain open and buy pandemic insurance when the inter-temporal correlation of pandemic losses is small (i.e., $p \leq q \leq \hat{q}$); businesses choose to close and do not buy pandemic insurance when the inter-temporal correlation is large (i.e., $\hat{q} < q \leq 1$). In period 2, businesses choose to open and buy pandemic insurance.

The intuition is as follows. If businesses close proactively in period 1, they will not buy pandemic insurance since the insurance does not cover losses due to proactive closure. Businesses have to bear the loss l themselves and have a low period-1 expected utility. As a tradeoff, the closure stops the spread of the pandemic, reduces the insurance premium in period 2, and thus results in a higher expected utility in period 2. If businesses choose to open in period 1, they will buy pandemic insurance, as we show in the equilibrium of the single-period model. The decision to remain open aggravates the pandemic, increases the insurance premium in period 2, and reduces the expected utility in period 2. A tradeoff thus exists between the expected utilities of the two periods. When the inter-temporal correlation is low, opening would have little effect on the spread of the pandemic and hence there would be a small decrease in period-2 expected utility but a large improvement in period-1 expected utility. Therefore, it is optimal for businesses to open in period 1 when the inter-temporal correlation is below a certain threshold, i.e., $p \leq q \leq \hat{q}$. When the inter-temporal correlation exceeds the threshold, i.e., $\hat{q} < q \leq 1$, it is optimal to close businesses proactively in period 1. In period 2, businesses consider the current-period expected utility only. Compared to closing the business and bearing the loss l themselves, it is always better for businesses to open and share the losses with the insurer by buying pandemic insurance.[26]

Corollary 1.1 suggests that neither private nor government reinsurance solve the market failure problem of pandemic insurance when the inter-temporal correlation is excessively high. Therefore, business lockdown policies are expected to keep the inter-temporal correlation of pandemic losses lower than the market failure threshold and are critical to the viability of any pandemic risk transfer solutions.

Corollary 1.2. ***Existence of pandemic insurance market***

In a pandemic insurance market with low competition, both private and government reinsurance improves the viability of pandemic insurance compared to the case of no reinsurance; however, government reinsurance can be less effective than private reinsurance:

$$\frac{\partial \hat{q}_p}{\partial \overline{K}} > 0, \ \frac{\partial \hat{q}}{\partial \overline{K}} > 0, \text{ and } \hat{q}_p > \hat{q}, \text{ when } \acute{\eta} < \eta \leq 1. \tag{31}$$

The intuition is as follows. In less competitive markets where $\acute{\eta} < \eta \leq 1$, reinsurance mainly

[26]Our results are expected to hold if we extend the model from two-period to multi-period. Businesses are more willing to close proactively in the early periods since they care more about their future expected utility and remain open in the late periods since the future expected utility becomes less important.

acts as a substitute for the insurer's capital and the saved capital cost reduces the insurer's minimum acceptable premium. The maximum willingness to pay makes businesses indifferent to two paths in the equilibrium: (i) open and buy insurance in period 1 and remain open in period 2; (ii) close in period 1 and open in period 2. Reinsurance improves the quality of pandemic insurance in both periods and thus increases the expected utilities of businesses on both paths. It is thus undetermined whether the maximum willingness to pay of businesses with reinsurance rises or not, compared to that without reinsurance. Therefore, the reduction in the minimum acceptable premium dominates and reinsurance narrows the gap between the minimum acceptable premium and the maximum willingness to pay compared to the case of no reinsurance. As a result, a larger reinsurance coverage $\overline{K}$ leads to a higher upper bound of inter-temporal correlation for the market existence in period 1. In other words, businesses are more likely to open in period 1. Corollary 1.2 suggests that, given any level of correlation among individual risks δ, reinsurance encourages businesses to open, i.e., the economy is more likely to keep running with pandemic reinsurance.[27]

Next, we compare the effects of government reinsurance and private reinsurance on the viability of the pandemic insurance market. In less competitive markets where $\acute{\eta} < \eta \leq 1$, the positive pandemic tax of government reinsurance decreases the expected utilities of businesses. If businesses open and buy the insurance in both periods, they have to pay the pandemic tax in both periods and thus face a greater reduction in expected utilities compared to closing in period 1 and reopening in period 2, where the pandemic tax does not exist in period 1 as the pandemic insurance market does not exist. Thus, opening in both periods is less attractive with government reinsurance than that with private reinsurance. In other words, businesses are less likely to open in period 1 with government reinsurance than with private reinsurance.[28]

Corollary 1.3. ***Improvement in expected social utility***

Government reinsurance improves the expected social utility compared to the cases of no reinsurance and private reinsurance:

$$V^e_{qn} < V^e_{qp} < V^e_{q}. \tag{32}$$

[27] In more competitive markets, reinsurance mainly plays the role of a safety buffer rather than saving the cost of capital and thus has a small impact on the insurer's minimum acceptable premium. Thus, it is undetermined whether a larger reinsurance coverage $\overline{K}$ increases or decreases the upper bound of inter-temporal correlation for market existence, i.e., $\frac{\partial \hat{q}_p}{\partial \overline{K}} \lesseqgtr 0$ and $\frac{\partial \hat{q}}{\partial \overline{K}} \lesseqgtr 0$ when $0 \leq \eta \leq \acute{\eta}$.

[28] In more competitive markets, the impact of pandemic tax on businesses' expected utility is undetermined between opening for two periods and closing in period 1, i.e., $\hat{q}_p \lesseqgtr \hat{q}$ when $0 \leq \eta \leq \acute{\eta}$.

The intuition is as follows. The inter-temporal correlation of pandemic losses does not affect the function of reinsurance—substituting for the insurer's holding capital at a lower cost. Thus, private reinsurance improves the expected social utility compared to no reinsurance. The inter-temporal correlation also does not affect the government's balance between catastrophe/pandemic taxes and reinsurance premiums to achieve the optimal expected social utility. Thus, as in Proposition 2, government reinsurance further improves the expected social utility compared to private reinsurance.

In reality, it is difficult for the insurer to distinguish business closure for precautionary purpose from that forced by the pandemic. Therefore, a moral hazard problem may arise when businesses can hide their intention of closure to maximize their inter-temporal expected utilities. To analyze this moral hazard problem, we consider the case that pandemic insurance indemnifies all losses caused by business closure.

Corollary 1.4. ***Moral hazard problem***

The viability of pandemic insurance is reduced, when the purpose of business closure is unobservable:

$$\hat{q}_{mh} < \hat{q}. \tag{33}$$

The intuition is as follows. With the moral hazard problem, the expected utility of businesses to close in period 1 increases because businesses can claim their losses from pandemic insurance regardless the real purpose of the close. Anticipating this moral hazard problem, the minimum acceptable premium of the insurer will increase. Given that businesses' maximum willingness to pay is unchanged, the market becomes harder to exist with moral hazard than without it.

5.3. Government-backed (Re)insurance Solutions for COVID-19 and Future Pandemics

Private pandemic risk protection remains largely unavailable in the market because pandemics are inherently uninsurable; "they are too widespread, too severe, and too unpredictable for the insurance industry to underwrite"(APCIA et al., 2020). Therefore, the insurance industry and some House Representatives are calling for public-private risk-sharing solutions to offer pandemic (re)insurance. PRIA and BCPP are two prominent and competing pandemic (re)insurance solutions under discussion in the U.S.. Both of them aim to support the pandemic risk transfer for business interruptions. One major difference between them is that PRIA proposes to establish a government reinsurance program and BCPP is a type of government-provided insurance. Our catastrophe risk-sharing model can thus be helpful to predict and to compare the outcomes of PRIA and BCPP.

PRIA was published the first draft on April 3, 2020, and proposed by Representative Carolyn Maloney and more than 20 co-sponsors on May 22 and 26 to the House and referred to the House Committee of Financial Services. PRIA proposes to establish a Pandemic Risk Reinsurance Program (PRRP hereafter) within the Department of the Treasury, under which private insurers and the federal government share the responsibility to pay claims for covered business interruption losses. Private insurers can voluntarily participate and participants are required to cover defined public health emergencies such as COVID-19 under their business interruption policies. To trigger the PRRP obligation, the aggregate industry insured losses should exceed $250 million. Once the PRRP has been triggered and participating insurers have paid the retention as 5% of their direct earned premiums during the preceding calendar year, the PRRP would pay 95% of the insured losses that exceed the insurer's retention. The total reinsurance coverage of PRRP is $750 billion based on the aggregate industry insured losses. PRIA is a government reinsurance program for pandemic risk-sharing that features public-private partnership.

The funding source of PRIA (PRRP) remains under discussion. In the version of April 3, PRIA requires the Treasury to charge a reinsurance premium to participating insurers (Maloney, 2020). The versions of May 22 and 26 strike this language and imply that funding will come from the taxpayers (Dawson, 2020; Maloney, 2020). Our model results suggest that there exists an optimal balance between the use of reinsurance premiums and of a catastrophe tax to fund a government reinsurance program. The premium-tax tradeoff raises individuals' demand for catastrophe insurance (with higher maximum willingness to pay) and makes the catastrophe risk-sharing market more effective (with higher catastrophe insurance product quality) as well as more efficient (with lower catastrophe insurance capital cost). We thus argue that PRIA should take advantage of this premium-tax tradeoff and adopt a mix of funding from participating insurers and taxpayers.

BCPP was proposed by three insurance industry trade groups, APCIA, NAMIC, and the Big "I" on May 21, 2020. It is a federal pandemic insurance for business interruption run by the Federal Emergency Management Agency (FEMA). Businesses can participate in the BCPP by purchasing federal revenue replacement assistance through a state-regulated insurance agent or carrier (as a sales agent). Premiums charged by BCPP is calculated as a percentage of the payroll of each business and applicable expenses for replacement. Businesses are allowed to choose a desired level of protection for three months' relief for up to 80% of payroll, employee benefits, and operating expenses. Once a federally declared public health emergency occurs, businesses would receive the federal revenue replacement assistance from FEMA. BCPP is a government insurance program fully funded by taxpayers and it is expected to fill the gap in the private insurance market.

Under PRIA, businesses decide whether to purchase pandemic insurance; the private insurer decides whether to participate in the PRRP, holds capital C_{PRIA}, and sets the pandemic insurance premium α_{PRIA} given that the market is not perfectly competitive. The PRRP collects the reinsurance premium M_{PRIA} from the private insurer and also funds the program with the tax subsidies T_{PRIA} to maximize the expected social utility V_{PRIA}, which is equal to the sum of businesses' expected utilities and the insurer's expected profit. The PRIA equilibrium is thus the same as the government reinsurance equilibrium.

BCPP, as a government insurance, can be seen as a degenerated government reinsurance case with $K = 0$, $\overline{K} = l$, and $M = \alpha$. Under BCPP, FEMA determines the price of pandemic insurance α_{BCPP} to maximize the expected social utility V_{BCPP}, which is businesses' expected utilities as the private market of pandemic insurance is negligible due to uninsurability. The budget constraint is $\alpha_{BCPP} \geq pl$. All businesses decide simultaneously whether to buy the insurance from BCPP. In the BCPP equilibrium, we can prove that the budget constraint is binding and the insurance price is equal to the expected loss, $\alpha^e_{BCPP} = pl$, and all businesses decide to buy pandemic insurance.

For the purpose of comparison, we assume that PRIA and BCPP have the same scope of coverage l and relax it afterwards. We derive the following four corollaries and the corresponding proofs are presented in Appendix D3.

Corollary 2.1. *Both PRIA and BCPP ensure the viability of pandemic risk transfer:*

$$\alpha^*_{PRIA}(C^e_{PRIA}, T^e_{PRIA}) > \underline{\alpha}_{PRIA}(C^e_{PRIA}, M^e_{PRIA}), \tag{34}$$

$$\alpha^*_{BCPP} > \underline{\alpha}_{BCPP}. \tag{35}$$

Both programs close the gap between the minimum acceptable premium of pandemic insurance and businesses' maximum willingness to pay. PRIA, as a government reinsurance program, ensures the existence of a private pandemic insurance market, according to Proposition 1. BCPP directly offers pandemic insurance at a fair price and therefore also ensures the viability of pandemic risk transfer.

Corollary 2.2. *PRIA is more efficient than BCPP in terms of expected social utility in a pandemic* *insurance market with low competition:*

$$V^e_{PRIA} > V^e_{BCPP}, \text{ when } \hat{\eta} < \eta \leq 1. \tag{36}$$

The intuition is as follows. The private insurer will have a positive expected profit with PRIA given that the private insurance market is not perfectly competitive; the private insurer will not participate in the pandemic insurance market and earns zero profit with BCPP. Businesses are always better off with BCPP, because it offers a zero-default insurance at a fair price. The comparison of expected social utility thus depends on the weight of the insurer's expected profit in the expected social utility with PRIA V^e_{PRIA}. When the private pandemic insurance market in the PRIA case is less competitive, the insurer's expected profit is high and thus drives the expected social utility with PRIA higher than that with BCPP.[29]

A government (re)insurance program faces a budget deficit when the covered losses exceed the program's accumulated reserves in a particular year t. The government has to borrow from other public or private financial resources though it can repay the loans with the program income in later periods. In period t, the probability of a budget deficit can be defined as $DP^e_{PRIA} \equiv Prob\left(I_{re}(x_t) > t(M^e_{PRIA} + T^e_{PRIA}) - \sum_{i=1}^{t-1} I_{re}(x_i)\right)$ and $DP^e_{BCPP} \equiv Prob\left(x_t l > t\alpha^e_{BCPP} - \sum_{i=1}^{t-1} x_i l\right)$, i.e., the probability that the covered losses exceed the accumulated reserves. The severity of a budget deficit is measured by the gap between the covered losses and the accumulated reserves, i.e., $DS^e_{PRIA} \equiv I_{re}(x_t) - \left[t(M^e_{PRIA} + T^e_{PRIA}) - \sum_{i=1}^{t-1} I_{re}(x_i)\right]$ and $DS^e_{BCPP} \equiv x_t l - \left(t\alpha^e_{BCPP} - \sum_{i=1}^{t-1} x_i l\right)$.

Corollary 2.3. *The probability and severity of PRIA's budget deficit are lower than BCPP's for pandemic risks with low event probability:*

$$DP^e_{PRIA} < DP^e_{BCPP} \text{ and } DS^e_{PRIA} < DS^e_{BCPP}, \text{ when } p < \frac{K}{tl}. \tag{37}$$

The intuition is as follows. BCPP covers the pandemic losses from the first dollar, while PRIA (PRRP) shares the pandemic risk with the private insurer and covers the losses between the insurer's retention and a reinsurance limit. The reserves of PRIA and BCPP are accumulated from the actuarially fair premium income in each period. When the pandemic probability p is small (i.e., $p < \frac{K}{tl}$), BCPP charges a price slightly higher than that of PRIA but covers much larger potential losses because PRIA (PRRP) is a non-proportional reinsurance and only covers the losses in the excess layer. The larger losses covered by BCPP are thus more likely to exceed the program

[29] When the private pandemic insurance market under PRIA is more competitive, businesses' expected utilities drive the expected social utility higher in the BCPP case than in the PRIA case, i.e., $V^e_{PRIA} \leq V^e_{BCPP}$, when $0 \leq \eta \leq \hat{\eta}$.

reserves, whereas PRIA can still pay for the losses with its reserves. In other words, PRIA has a lower probability of budget deficit.[30] The larger losses covered by BCPP also result in a larger gap between the covered losses and the program reserves, i.e., PRIA's budget deficit would be smaller.[31]

In practice, the pandemic risk market features low competition (few insurers) and low event probability (once every 10 to 20 years for a region). Therefore, PRIA is likely to be more efficient and more financially sustainable than BCPP. Previous analyses assume that PRIA and BCPP have the same scope of coverage. In reality, PRIA (PRRP) and its supported pandemic insurance cover businesses' income losses including both operating expenses and profit losses, while BCPP covers only the former. The broader coverage of PRIA will further improve businesses' expected utilities. Therefore, PRIA will be more efficient than BCPP considering their difference in coverage. PRIA may increase the default probability of private insurers offering pandemic coverage in a less competitive environment according to Proposition 3. Thus, keeping solvency regulation standards high is critical to the success of PRIA. A solvency-based entry requirement might be expected for private insurers to participate in the PRRP.

Previous analyses on PRIA and BCPP assume away the inter-temporal correlation of pandemic losses and its associated moral hazard problem. We show in the following corollary that incorporating them weakens the conclusion in Corollary 2.1 but does not change the qualitative results of the comparison between PRIA and BCPP in Corollaries 2.2 and 2.3.

Corollary 2.4. *PRIA is more effective than BCPP in term of in terms of supporting the pandemic risk transfer, when pandemic losses are inter-temporally correlated and the purpose of business closure is unobservable:*

$$\hat{q}_{BCPP} < \hat{q}_{PRIA}. \tag{38}$$

The intuition is as follows. In the two-period framework that allows businesses to hide their purpose of closure (i.e., the moral hazard is present), businesses tend to close proactively in period 1 and buy the government-provided pandemic insurance (i.e., BCPP) to fully recover their losses. Thus, in the equilibrium, BCPP cannot be available due to the moral hazard problem. In the PRIA

[30] For example, an insurer's retention is 0.4 billion and a 1 billion loss event occurs. PRRP would bear 0.6 billion of the losses and BCPP would bear 1 billion. When the pandemic probability p is small and the premiums are also small relative to the coverage, the accumulated reserves are less likely to fill such a large difference in coverage, and therefore BCPP is more likely to have a budget deficit.

[31] The relationship is undetermined for high-probability risks with $p \geq \frac{K}{tl}$.

case, businesses tend to open in period 1 as the their expected losses are smaller if they open than if they close. This is because business closure will result in a larger probability of the private insurer default and a larger default residual loss borne by the businesses. Therefore, PRIA is more effective than BCPP to support the pandemic risk transfer.

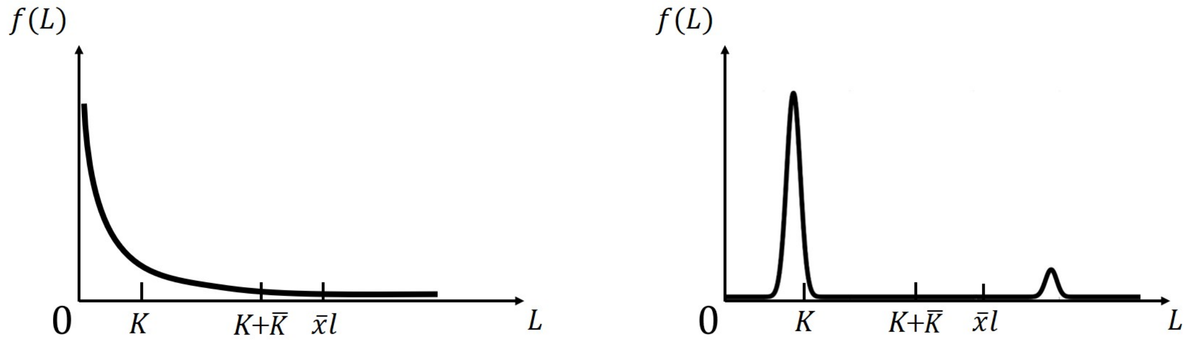
6. Discussion: Alternative Government Intervention Policy

In this section, we compare our government reinsurance program with two alternative government intervention policies that are (i) ex-post government catastrophe relief program and (ii) solvency regulation. A formal proof for the comparison results is provided in Appendices E1 and E2, respectively.

6.1. Ex-post catastrophe relief program

Charpentier and Le Maux (2014) develop a model to analyze an ex-post government catastrophe relief program (hereafter the GRel case, also called disaster aid program). In their model, the government charges a post-catastrophe tax $T(x)$ on each individual in a region to cover the residual loss when a monopolistic insurer defaults due to a catastrophe event. Thus, the ex-post tax depends on the share of catastrophe-hit population x, that is $T(x) = xl - (\alpha_{CL} + C_{CL})$ if the insurer is insolvent and $T(x) = 0$ if it is solvent. There is no reinsurance in the GRel case (i.e., $\overline{K} = M = 0$). In their equilibrium, the insurer sets the catastrophe insurance premium α^*_{CL} at each individual's maximum willingness to pay given exogenous zero-cost capital C_{CL} and a monopolistic catastrophe insurance market. To create a fair comparison, we add the cost of capital rC_{CL} to the insurer's expected profit equation in the GRel case.

Government reinsurance and ex-post catastrophe relief programs function at different phases in the development of catastrophe risks: the government reinsurance shares losses before the insurer defaults and the ex-post catastrophe relief program covers the residual loss after the insurer defaults. Therefore, the government reinsurance program is more efficient if the catastrophe risk is moderately heavy-tailed such that the catastrophe losses L are more concentrated in the range covered by the CAT XL reinsurance between K and $K + \overline{K}$, for example, a Pareto distribution shown in Figure 7(a). The catastrophe relief program is more efficient if the catastrophe risk are extremely heavy-tailed such that the residual loss after insurer default (i.e., the losses beyond $\bar{x}l$) are large, for example, the distribution used by Charpentier and Le Maux (2014) in Figure 7(b).



(a) Pareto Distribution (b) CL Distribution

Figure 7: Moderate vs. Extreme Heavy-Tail Catastrophe Risk Distributions

The combination of government reinsurance and catastrophe relief programs (the "RR" case, hereafter) will, however, lead to a definite improvement in the expected social utility. In the RR equilibrium, the government covers both the catastrophe losses under the reinsurance and the default losses of the insurer. Thus, individuals will have expected utility gains from the free-of-default RR program and the insurer can further saves capital cost by charging higher premiums when individuals are willing to pay more for the no-default insurance. Depending on the market structure, individuals and the insurer share the additional benefits of the combined program. Our results suggest that establishing a formal government reinsurance program with ex-ante catastrophe tax and reinsurance premium is efficient and practical in addition to the ad-hoc post-catastrophe relief that a modern government is always expected to offer.

Given a positive capital cost, government reinsurance ensures the existence of the private catastrophe insurance market, but the catastrophe relief program cannot. Moreover, government reinsurance can take the advantage of the tradeoff between catastrophe tax and reinsurance premium to further improve the expected social utility, while the catastrophe relief program cannot because the ex-post catastrophe tax in the GRel case is determined by the amount of residual losses instead of the government's optimal choice. A practical concern also arises with charging ex-post tax to catastrophe victims because they are usually experiencing financial difficulties. Our ex-ante catastrophe tax design addresses this concern. The disadvantage of government reinsurance lies that it may increase the insurer's default probability when competition is insufficient, while the catastrophe relief program can always reduce the insurer's default probability.

6.2. Solvency regulation

Solvency capital regulation is a prevalent tool of governments and regulators to limit the default probability of insurers. Bernard and Tian (2009) define the solvency regulation as follows:

$$Prob\left[xl > \alpha + C + R(\overline{K} - M)\right] \leq \theta, \tag{39}$$

where θ ($0 < \theta < 1$) is the maximum acceptable default probability of a solvency regulation. In the regulated GR case, the solvency regulation is a constraint on the insurer. Other decision process remains unchanged compared to the unregulated GR case. Define θ^e the insurer's default probability in the unregulated GR equilibrium. When the solvency regulation requires a default probability higher than or equal to θ^e, the regulation is not tight and the regulated GR equilibrium is the same as the unregulated GR equilibrium. When the solvency regulation requires a default probability lower than θ^e, the insurer will hold capital at the minimum level that meets the regulatory requirement $C_s^e = \bar{x}_s l - \alpha_s^e + M_s^e - \overline{K}$, where $\bar{x}_s \equiv VaR_x(\theta)$ (i.e., $Prob(x \geq \bar{x}_s) = \theta$) is exogenous. α_s^e and M_s^e are the insurance premium and reinsurance premium in the regulated GR equilibrium.

The goals of government reinsurance and solvency regulation are different. The former aims to improve the expected social utility and the latter to reduce the insurer's default probability. Thus, naturally, solvency regulation introduces a tradeoff between the regulatory target default probability and the expected social utility. As shown in Figure 8, when the solvency regulation requires the insurer to hold more capital than the unregulated optimal capital (i.e., $0 < \theta < \theta^e$), the extra capital causes a higher cost for the insurer, decreases the insurer's expected profit, and thus decreases the expected social utility. The insurer may also need more tax subsidy to reinsurance premium when it cannot bear all the increased cost of capital and faces competitive pressure on expected profit. The additional tax beyond the unregulated optimal level will also decrease individuals' expected utilities, and hence the expected social utility. The optimal expected social utility in the regulated GR case is equal to that in the unregulated case when the solvency regulation is not tight (i.e., $\theta \geq \theta^e$). To achieve a lower default probability than θ^e, the government can impose a tight solvency regulation at the expense of lower expected social utility.

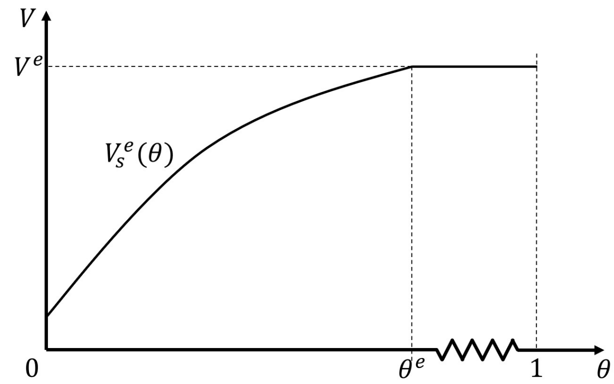


Figure 8: Tradeoff between Target Default Probability and Expected Social Utility

Note: $V_s^e(\theta)$ (V^e) is the optimal expected social utility in the regulated (unregulated) GR equilibrium. When $\theta < \theta^e$, $V_s^e(\theta)$ is increasing in θ. When $\theta \geq \theta^e$, $V_s^e(\theta)$ is equal to V^e.

This tradeoff is particularly important in the catastrophe risk market where insurance and risk diversification are insufficient (Gollier, 2008). Solvency regulation alone may exacerbates the underinsurance problem in a catastrophe insurance market because it imposes additional capital cost for the insurer and further decreases the insurer's profitability. A combination of solvency regulation and government reinsurance can maximize the expected social utility at an acceptable level of default probability.

7. Conclusion

We develop a dynamic game model to investigate the equilibrium of a catastrophe risk market with government reinsurance. To our best knowledge, no theoretical framework has been developed to characterize the catastrophe market equilibrium involving three decision makers in both the primary insurance and reinsurance markets. Our model for efficient catastrophe risk-sharing fills this gap by enabling individuals, the private insurer, and the government to derive their optimal pricing, capital, and purchasing decisions in one equilibrium. Our model is original in that (i) it introduces the tradeoff of a centralized agency to fund the reinsurance program between catastrophe taxes charged from individuals and reinsurance premiums charged from the insurer; this tradeoff improves the social welfare, and that (ii) it identifies new impact channels of government intervention on the efficiency of catastrophe risk-sharing.

Our public-private risk-sharing model offers a useful framework to analyze the ongoing catastrophe of the COVID-19 pandemic and its (re)insurance solutions. Our model calibrations based on various COVID-19 scenarios show that government reinsurance can improve the efficiency of pandemic risk-sharing compared to the cases of no reinsurance and private reinsurance. However, the efficiency gains are traded-off with a "pandemic tax" that decreases individuals' expected utilities in a low-competition pandemic insurance market and are insufficient to compensate the welfare losses due to massively widespread pandemic. Different from many natural catastrophes, the pandemic losses are inter-temporally correlated and therefore introduce a tradeoff between economic development (business open) and pandemic spread. Both private and government reinsurance can improve the viability of pandemic insurance and therefore support businesses to open, which is, however, at the cost of continuing the pandemic spread. When the inter-temporal correlation is excessively large, neither government nor private reinsurance can help to sustain the pandemic insurance market.

The model predictions and calibrations yield important public policy implications on the risk management of the COVID-19 and future pandemics. A public-private partnership in pandemic risk-sharing, for example, the PRIA and BCPP discussed in the U.S., is necessary to ensure the viability of pandemic risk transfer. Our analyses favor the government reinsurance program (PRIA) as it is more efficient in terms of the expected social utility and more financially sustainable than the government insurance program (BCPP) in a low-competition and low-frequency pandemic insurance environment. Taking advantage of the tradeoff between reinsurance premium and pandemic tax to fund the government reinsurance program will further improve the social welfare under PRIA. Moreover, any government pandemic reinsurance program should be coupled with social distancing measures to control for the pandemic spread, with business lockdown policies to reduce the inter-temporal correlation of pandemic losses, and with anti-monopoly policies to balance the interests of individuals and insurers.

The equilibrium in the catastrophe risk market with government reinsurance is a set of optimal decisions of the individuals, the private insurer, and the government: (i) All individuals buy the catastrophe insurance, (ii) the private insurer earns a nonnegative expected profit from offering the full coverage and buying the government reinsurance, and (iii) the government funds the reinsurance program by charging reinsurance premiums from the insurer and ex-ante catastrophe taxes from all individuals exposed to the catastrophe risk. The private catastrophe insurance market fails to exist when both risk correlation and capital cost exceed their respective thresholds. The results justify a third-party risk-sharing mechanism in a catastrophe risk market. Compared with private reinsurance, government reinsurance achieves the best balance between individuals' expected utilities and the insurer's expected profit and therefore improves the social welfare (defined as the expected social utility). In less competitive catastrophe insurance markets, however, government reinsurance may reduce the expected utilities of individuals and increase the default probability of the insurer.

Our results add new insights to the economics of catastrophe risk-sharing. The advantages of government reinsurance over private reinsurance manifest via the product quality and capital cost channels, which capture the strategic behavior changes of existing market players. On the one hand, the catastrophe tax subsidizes the reinsurance premium; the saved reinsurance premium enables the insurer to hold more costly capital, which improves the product quality, and thus increases individuals' expected utilities (i.e., the product quality channel). On the other hand, the catastrophe tax increases individuals' maximum willingness to pay for and thus the total contribution to catastrophe risk transfer, which offsets the insurer's incentive to hold more capital for higher premium, saves its capital cost, and thus increases the insurer's expected profit (i.e., the capital cost channel). Both channels transmit a positive impact of government reinsurance on the expected social utility, but have opposite impacts on the insurer's default probability. Through the product quality (capital cost) channel, the government reinsurance decreases (increases) the insurer's default probability. The strengths of the two channels wane and wax depending on the market structure. As the insurance market becomes more competitive, the product quality channel becomes stronger, but the capital cost channel weakens. The product quality channel shuts down in a monopolistic market and the capital cost channel shuts down in a perfectly competitive market.

The discovery of the equilibrium and its two channels offers new managerial insights to catastrophe risk management. By sharing the catastrophe risks of individuals and the private insurer, government reinsurance improves the catastrophe risk market towards a more willing-to-participate and more capital-cost-efficient state. The government reinsurance is applicable to broad types of catastrophe risks and to various layers of reinsurance retention and limit. The pricing of government reinsurance is risk-based, affordable, long-term wise breakeven without external financial resources, and sustainable with certain loadings of operational costs and interest rates. Our results also suggest that the government reinsurance should always be coupled with the anti-monopoly policies. Although large insurers are better at catastrophe risk diversification, competition in a catastrophe insurance market has its unique values and should be encouraged for two reasons: (i) competition balances individuals' expected utilities and the insurer's expected profit to achieve the Pareto improvement; (ii) competition mitigates the problem of increasing default probability of the insurer due to the government reinsurance.

We frame the model in the terminology of catastrophe/pandemic risk markets. It can also be used as a general framework for analyzing risk-sharing decisions with a public-private partnership nature; that is, three parties—the public, the private, and consumers—have their own objectives and risk-sharing decisions to make. For example, the reserve-bailout mechanism of International Monetary Fund can be seen as a type of reinsurance provided by a centralized agency to cover the systemic risk of economic/financial crisis where the national/local government plays the role of insurer. The concept of government reinsurance may offer a new perspective to revisit the social insurance program. Conventionally, public pension and social health insurance provide coverage from the ground risk layer, so as the private pension and private health insurance. Can social insurance programs concentrate the limited resources on covering the undiversified catastrophe layer of longevity and pandemic risks by offering reinsurance to support the private insurance market? Our public-private risk-sharing model offers a new framework to analyze these questions.

Our model assumes an exogenous capital cost that reflects the expected return of investors. Future research might consider to connect the insurance market and the capital market by linking the degree of individuals' risk aversion and the insurer's shareholders' expected return to bear the catastrophe risks. Moreover, this paper focuses on comparing government reinsurance with private reinsurance and, therefore, do not consider their competition in one catastrophe risk market. We leave these questions for future research.

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Public-Private Catastrophe Risk-Sharing: Theory and Application to COVID-19

All Appendices are intended for review and for online publishing as supplementary materials. The Appendices are ordered according to where they are first referenced in the main text. Appendix A proves the equilibrium of our catastrophe risk-sharing model. Appendix B proves the propositions. Appendix C proves the impact channels and pricing of government reinsurance. Appendix D presents proof and additional results for the COVID-19 applications. Appendix E proves the alternative government intervention policies.

List of Appendices

Appendix A: Proof of Equilibriums

Appendix A1: Proof of Equilibrium with No Reinsurance

The NR case is a degenerated GR case with $T = K = \overline{K} = M = 0$. There are two players in the game: individuals and the insurer. We solve the game with backward induction starting from individuals' purchasing decisions and then to the insurer's optimal pricing and capital decisions.

When the risk correlation and the unit cost of capital exceed their respective thresholds, i.e., $\delta > \underline{\delta}$ and $r > \underline{r}$, the market fails to exist as shown in Proposition 1. The insurer earns zero expected profit $\Pi_n^e = 0$ and individuals end up with uninsured expected utility $U_{in}^e = pU(-l)$. The expected social utility is thus $V_n^e = U_{in}^e$.

When the private catastrophe insurance market exists, i.e., $0 \leq \delta \leq \underline{\delta}$ or $0 \leq r \leq \underline{r}$, we prove that all individuals purchase the catastrophe insurance, i.e., $\lambda_n^e = 1$ and derive the insurer's optimal decisions in different market structures, respectively.

Suppose to the contrary that in the equilibrium, the insurer sells the catastrophe insurance to λ_{n1} $(0 < \lambda_{n1} < 1)$ individuals with insurance premium α_{n1} and holding capital C_{n1}. The insurer's expected profit is non-negative:

$$\Pi_n(\lambda_{n1}, \alpha_{n1}, C_{n1}) = \int_0^{\bar{x}_{n1}} (\lambda_{n1}\alpha_{n1} - \lambda_{n1}xl) f(x)dx - [1 - F(\bar{x}_{n1})]C_{n1} - rC_{n1} \geq 0, \tag{A1-1}$$

where $\bar{x}_{n1} = \frac{1}{\lambda_{n1}l}(\lambda_{n1}\alpha_{n1} + C_{n1})$. For those insured individuals, their insured expected utility is no less than their uninsured expected utility:

$$\begin{aligned} U_{in}(\lambda_{n1}, \alpha_{n1}, C_{n1}) &= \int_0^{\bar{x}_{n1}} xU(-\alpha_{n1}) f(x)dx + \int_{\bar{x}_{n1}}^1 xU(-\alpha_{n1} - l + \frac{\bar{x}_{n1}}{x}l) f(x)dx + \int_0^1 (1-x)U(-\alpha_{n1}) f(x)dx \\ &\geq pU(-l). \end{aligned} \tag{A1-2}$$

Alternatively, we can prove that there exists an insurance premium α_{n1} and capital $\frac{C_{n1}}{\lambda_{n1}}$ such that *all* individuals are willing to purchase the catastrophe insurance as the insured expected utility is higher than uninsured utility and that the insurer achieves higher expected profit by selling insurance to all individuals:

$$\begin{aligned} U_{in}(\lambda = 1, \alpha_{n1}, \frac{C_{n1}}{\lambda_{n1}}) &= \int_0^{\bar{x}_{n2}} xU(-\alpha_{n1}) f(x)dx + \int_{\bar{x}_{n1}}^1 xU(-\alpha_{n1} - l + \frac{\bar{x}_{n2}}{x}l) f(x)dx + \int_0^1 (1-x)U(-\alpha_{n1}) f(x)dx \\ &= U_{in}(\lambda_{n1}, \alpha_{n1}, C_{n1}) \\ &\geq pU(-l), \end{aligned} \tag{A1-3}$$

where the first equality is the definition of insured expected utility; the second equality follows from $\bar{x}_{n2} = \frac{1}{l}(\alpha_{n1} + \frac{C_{n1}}{\lambda_{n1}}) = \bar{x}_{n1}$; and the inequality follows from Eq.(A1-2).

$$\begin{aligned}\Pi_n(\lambda = 1, \alpha_{n1}, \frac{C_{n1}}{\lambda_{n1}}) &= \int_0^{\bar{x}_{n2}} (\alpha_{n1} - xl) f(x) dx - [1 - F(\bar{x}_{n2})] \frac{C_{n1}}{\lambda_{n1}} - r\frac{C_{n1}}{\lambda_{n1}} \\ &= \frac{1}{\lambda_{n1}} \Pi_n(\lambda_{n1}, \alpha_{n1}, C_{n1}) \\ &> \Pi_n(\lambda_{n1}, \alpha_{n1}, C_{n1}),\end{aligned} \tag{A1-4}$$

where the first equality is the definition of the insurer's expected profit; the second equality follows from $\bar{x}_{n2} = \bar{x}_{n1}$; and the inequality follows from $0 < \lambda_{n1} < 1$.

In conclusion, both the insurer and individuals have incentives to insure the rest $1 - \lambda_{n1}$ of individuals. Thus, in the equilibrium, i.e., $\lambda_n^e = 1$.

Next, we prove the optimal decisions of the insurer in the NR equilibrium. Note that at the maximum willingness to pay $\alpha_n^*(C_n)$, individuals are indifferent between buying the catastrophe insurance or not:

$$\int_0^{\bar{x}_{n3}} xU(-\alpha_n^*(C_n)) f(x) dx + \int_{\bar{x}_{n3}}^1 xU\left(-\alpha_n^*(C_n) - l + \frac{\bar{x}_{n3}}{x} l\right) f(x) dx + \int_0^1 (1-x) U(-\alpha_n^*(C_n)) f(x) dx = pU(-l), \tag{A1-5}$$

where $\bar{x}_{n3} = \frac{1}{l}(\alpha_n^*(C_n) + C_n)$.

In a monopolistic market where $\eta = 1$, the insurer decides the insurance premium α_n and its capital C_n to maximize its expected profit, i.e.,$\max_{\alpha_n, C_n} \Pi_n(\alpha_n, C_n) = \int_0^{\bar{x}_n} (\alpha_n - xl) f(x) dx - [1 - F(\bar{x}_n)] C_n - rC_n$. Given any C_n, the insurer's expected profit $\Pi_n(\alpha_n, C_n)$ is increasing in α_n and thus the insurer sets the premium at individuals' maximum willingness to pay $\alpha_n^*(C_n)$. The optimal capital C_n^* satisfies:

$$\frac{d}{dC_n} \Pi_n(\alpha_n^*(C_n^*), C_n^*) = F(\bar{x}_n^*) \frac{d}{dC_n} \alpha_n^*(C_n^*) - [1 - F(\bar{x}_n^*) + r] = 0, \tag{A1-6}$$

where $\bar{x}_n^* = \frac{1}{l}(\alpha_n^*(C_n^*) + C_n^*)$. Thus, The optimal insurance premium α_n^* is $\alpha_n^* = \alpha_n^*(C_n^*)$. Therefore, the monopolistic insurer's expected profit in the equilibrium is:

$$\Pi_n^* = \int_0^{\bar{x}_n^*} (\alpha_n^* - xl) f(x) dx - [1 - F(\bar{x}_n^*)] C_n^* - rC_n^*. \tag{A1-7}$$

In a perfectly competitive market where $\eta = 0$, the insurer offers the market price $\widetilde{\alpha}$ to individuals. Its optimal capital C_n^{**} can be derived from the zero expected profit condition:

$$\int_0^{\bar{x}_n^{**}} (\widetilde{\alpha} - xl) f(x) dx - [1 - F(\bar{x}_n^{**})] C_n^{**} - rC_n^{**} = 0, \tag{A1-8}$$

where $\bar{x}_n^{**} = \frac{1}{l}(\widetilde{\alpha} + C_n^{**})$.

In markets with imperfect competition where $0 < \eta < 1$, the insurer decides the insurance premium α_n and its capital C_n to maximize its expected profit with constraints:

$$\begin{aligned} \max_{\alpha_n, C_n} \Pi_n(\alpha_n, C_n) &= \int_0^{\bar{x}_n} (\alpha_n - xl) f(x) dx - [1 - F(\bar{x}_n)] C_n - rC_n. \\ s.t. \alpha_n &= \eta \alpha_n^*(C_n) + (1-\eta)\widetilde{\alpha}, \\ \Pi_n(\alpha_n, C_n) &\leq \eta \Pi_n^*, \end{aligned} \tag{A1-9}$$

where the first constraint limits the market pricing power and the second constraint limits the profitability due to competitive pressure. In the equilibrium, the optimal premium income capital $C^e_{premium,n}(\eta)$ maximizes the insurer's expected profit without considering the profitability constraint:

$$\begin{aligned} &\frac{d}{dC_n} \Pi_n(\eta \alpha_n^*(C^e_{premium,n}(\eta)) + (1-\eta)\widetilde{\alpha}, C^e_{premium,n}(\eta)) \\ &= \eta F(\bar{x}_{n4}) \frac{d}{dC_n} \alpha_n^*(C^e_{premium,n}(\eta)) - [1 - F(\bar{x}_{n4}) + r] = 0, \end{aligned} \tag{A1-10}$$

where $\bar{x}_{n4} = \frac{1}{l}[\alpha_n^*(C^e_{premium,n}(\eta)) + C^e_{premium,n}(\eta)]$. The optimal product quality capital $C^e_{quality,n}(\eta)$ can then be derived from the profitability constraint:

$$\int_0^{\bar{x}_n^e} [\alpha_n^e(\eta) - xl] f(x) dx - [1 - F(\bar{x}_n^e) + r][C^e_{quality,n}(\eta) + C^e_{premium,n}(\eta)] = \eta \Pi_n^*, \tag{A1-11}$$

where $\bar{x}_n^e = \frac{1}{l}[\alpha_n^e(\eta) + C^e_{quality,n}(\eta) + C^e_{premium,n}(\eta)]$. The optimal capital $C_n^e(\eta)$ is the sum of the product quality capital and the premium income capital, i.e., $C_n^e(\eta) = C^e_{quality,n}(\eta) + C^e_{premium,n}(\eta)$. The optimal insurance premium $\alpha_n^e(\eta)$ equals to the weighted sum of individuals' maximum willingness to pay and the exogenous market price, i.e., $\alpha_n^e(\eta) = \eta \alpha_n^*(C_n^e(\eta)) + (1-\eta)\widetilde{\alpha}$. When $\eta = 1$ ($\eta = 0$), we have $\alpha_n^e(1) = \alpha_n^*$ ($\alpha_n^e(0) = \widetilde{\alpha}$) and $C_n^e(1) = C_n^*$ ($C_n^e(0) = C_n^{**}$).

Therefore, each individual's expected utility, the insurer's expected profit, and the expected social utility are as follows:

$$U_{in}^e = \int_0^{\bar{x}_n^e} xU(-\alpha_n^e(\eta)) f(x) dx + \int_{\bar{x}_n^e}^1 xU(-\alpha_n^e(\eta) - l + \frac{\bar{x}_n^e}{x} l) f(x) dx + \int_0^1 (1-x) U(-\alpha_n^e(\eta)) f(x) dx, \tag{A1-12}$$

$$\Pi_n^e = \eta \Pi_n^*, \tag{A1-13}$$

$$V_n^e = U_{in}^e + \Pi_n^e. \tag{A1-14}$$

Appendix A2: Proof of Equilibrium with Private Reinsurance

The PR case is a degenerated GR case with $T = 0$. We first prove that all individuals purchase the catastrophe insurance in the equilibrium, i.e., $\lambda_p^e = 1$ and then derive the insurer's optimal pricing, capital, and reinsurance decisions.

$\lambda_p^e = 1$ follows from two steps: (i) When the insurer does not buy reinsurance, all individuals purchase the catastrophe insurance, which has been proved in Appendix A1; (ii) Buying reinsurance with fair premium does not decrease the insured population. Buying reinsurance increases the insurer's expected profit, motivating the insurer to offer insurance to more individuals. Buying reinsurance improves the insurance quality and thus individuals are also more willing to buy catastrophe insurance in the equilibrium. Thus, buying reinsurance does not decrease the insured population.

In the equilibrium, the insurer purchases the private reinsurance $R_p^e = 1$ because given any α_p and C_p, the insurer always earns a higher expected profit by buying reinsurance:

$$\begin{aligned}
&\Pi_p(\alpha_p, C_p - (\overline{K} - M_p), R_p = 1) \\
=&\int_0^{\bar{x}_{p1}} [\alpha_p - M_p - xl + I_{re}(x)] f(x)dx - [1 - F(\bar{x}_{p1})][C_p - (\overline{K} - M_p)] - r[C_p - (\overline{K} - M_p)] \\
=&\int_0^{\bar{x}_{p1}} (\alpha_p - xl) f(x)dx - [1 - F(\bar{x}_{p1})]C_p - rC_p + r(\overline{K} - M_p) \\
>&\Pi(\alpha_p, C_p, R_p = 0),
\end{aligned} \tag{A2-1}$$

where $\bar{x}_{p1} = \frac{1}{l}(\alpha_p + C_p)$.

Note that at the maximum willingness to pay $\alpha_p^*(C_p)$, individuals are indifferent between buying insurance or not:

$$\int_0^{\bar{x}_{p2}} xU(-\alpha_p^*(C_p))f(x)dx + \int_{\bar{x}_{p2}}^1 xU\left(-\alpha_p^*(C_p) - l + \frac{\bar{x}_{p2}}{x}l\right) f(x)dx + \int_0^1 (1-x)U(-\alpha_p^*(C_p))f(x)dx = pU(-l), \tag{A2-2}$$

where $\bar{x}_{p2} = \frac{1}{l}[\alpha_p^*(C_p) + C_p + \overline{K} - M_p]$.

We derive the insurer's optimal pricing and capital decisions in different market structures, respectively. In a monopolistic market where $\eta = 1$, the insurer decides the insurance premium α_p and its capital C_p to maximize its expected profit, i.e., $\max_{\alpha_p, C_p} \Pi_p(\alpha_p, C_p, R_p^e = 1) = \int_0^{\bar{x}_p} [\alpha_p - M_p - xl + I_{re}(x)] f(x)dx - [1 - F(\bar{x}_p)]C_p - rC_p$. Given any C_p, the insurer's expected profit $\Pi_p(\alpha_p, C_p, R_p^e = 1)$ is increasing in α_p and thus the insurer sets the insurance premium at individuals' maximum willingness to pay $\alpha_p^*(C_p)$. The optimal capital C_p^* satisfies:

$$\frac{d}{dC_p}\Pi_p(\alpha_p^*(C_p), C_p, R_p^e = 1) = F(\bar{x}_p^*)\frac{d}{dC_p}\alpha_p^*(C_p^*) - [1 - F(\bar{x}_p^*) + r] = 0, \tag{A2-3}$$

where $\bar{x}_p^* = \frac{1}{l}[\alpha_p^*(C_p^*) + C_p^* + \overline{K} - M_p]$. Thus, the optimal insurance premium α_p^* is $\alpha_p^* = \alpha_p^*(C_p^*)$.

Therefore, the insurer's expected profit in the equilibrium is:

$$\Pi_p^* = \int_0^{\bar{x}_p^*} \left[\alpha_p^* - M_p - xl + I_{re}(x)\right] f(x)dx - [1 - F(\bar{x}_p^*)]C_p^* - rC_p^*. \tag{A2-4}$$

In a perfectly competitive market where $\eta = 0$, the insurer offers the market price $\widetilde{\alpha}$ to individuals. Its optimal capital C_p^{**} can be derived from the zero expected profit condition:

$$\int_0^{\bar{x}_p^{**}} [\widetilde{\alpha} - M_p - xl + I_{re}(x)] f(x)dx - [1 - F(\bar{x}_p^{**})]C_p^{**} - rC_p^{**} = 0, \tag{A2-5}$$

where $\bar{x}_p^{**} = \frac{1}{l}[\widetilde{\alpha} + C_p^{**} + \overline{K} - M_p]$.

In markets with imperfect competition where $0 < \eta < 1$, the insurer decides the insurance premium α_p and its capital C_p to maximize its expected profit with two constraints: (i) limited market pricing power constraint, i.e., $\alpha_p = \eta\alpha_p^*(C_p) + (1-\eta)\widetilde{\alpha}$ and (ii) profitability constraint, i.e., $\Pi_p(\alpha_p, C_p, R_p^e = 1) \leq \eta\Pi_p^*$. In the equilibrium, the optimal capital $C_p^e(\eta)$ is the sum of the optimal premium income capital and the optimal product quality capital, i.e., $C_p^e(\eta) = C_{premium,p}^e(\eta) + C_{quality,p}^e(\eta)$. The optimal premium income capital $C_{premium,p}^e(\eta)$ maximizes the insurer's expected profit without any profitability constraint:

$$\eta F(\bar{x}_{p3}) \frac{d}{dC_p} \alpha_p^*(C_{premium,p}^e(\eta)) - [1 - F(\bar{x}_{p3}) + r] = 0, \tag{A2-6}$$

where $\bar{x}_{p3} = \frac{1}{l}[\alpha_p^*(C_{premium,p}^e(\eta)) + C_{premium,p}^e(\eta) + \overline{K} - M_p]$. The optimal product quality capital $C_{quality,p}^e(\eta)$ is derived from the profitability constraint:

$$\int_0^{\bar{x}_p^e} \left[\alpha_p^e(\eta) - M_p - xl + I_{re}(x)\right] f(x)dx - [1 - F(\bar{x}_p^e) + r][C_{quality,p}^e(\eta) + C_{premium,p}^e(\eta)] = \eta\Pi_p^*, \tag{A2-7}$$

where $\bar{x}_p^e = \frac{1}{l}[\alpha_p^e(\eta) + C^e(\eta) + \overline{K} - M_p]$. The optimal insurance premium $\alpha_p^e(\eta)$ is at the weighted sum of individuals' maximum willingness to pay and the exogenous market price $\alpha_p^e(\eta) = \eta\alpha_p^*(C_p^e(\eta)) + (1-\eta)\widetilde{\alpha}$. When $\eta = 1$ ($\eta = 0$), we have $\alpha_p^e(1) = \alpha_p^*$ ($\alpha_p^e(0) = \widetilde{\alpha}$), $C_p^e(1) = C_p^*$ ($C_p^e(0) = C_p^{**}$).

Therefore, each individual's expected utility, the insurer's expected profit, and the expected social utility are as follows:

$$U_{ip}^e = \int_0^{\bar{x}_p^e} xU(-\alpha_p^e(\eta))f(x)dx + \int_{\bar{x}_p^e}^1 xU(-\alpha_p^e(\eta) - l + \frac{\bar{x}_p^e}{x}l)f(x)dx + \int_0^1 (1-x)U(-\alpha_p^e(\eta))f(x)dx, \tag{A2-8}$$

$$\Pi_p^e = \eta\Pi_p^*, \tag{A2-9}$$

$$V_p^e = U_{ip}^e + \Pi_p^e. \tag{A2-10}$$

Appendix A3: Proof of Equilibrium with Government Reinsurance

We prove the GR equilibrium in three steps with backward induction. We start from the last decision maker—the individuals. Then we derive the insurer's optimal decisions under different market structures. Last, we come to the government's optimal decisions.

Individuals' Decision

All individuals purchase the catastrophe insurance and the insurer is willing to offer insurance to all individuals in the GR equilibrium. The proofs are the same as those for the NR and PR equilibriums and documented in Appendices A1 and A2.

Insurer's Decisions

The insurer purchases the government reinsurance $R^e = 1$ because given any α, C, T, and $M = E[I_{re}(x)] - T$, the insurer always earns a higher expected profit by buying reinsurance:

$$\Pi(\alpha, C - (\overline{K} - M), R = 1, M) = \Pi(\alpha, C, R = 0, M) + T^e + r(\overline{K} - M) > \Pi(\alpha, C, R = 0, M). \tag{A3-1}$$

Note that at the maximum willingness to pay $\alpha^*(C,T)$, individuals are indifferent between buying catastrophe insurance or not:

$$\begin{aligned}&\int_0^{\overline{x}_1} xU\left(-\alpha^*(C,T) - T\right) f(x)dx + \int_{\overline{x}_1}^1 xU\left(-\alpha^*(C,T) - T - l + \frac{\overline{x}_1}{x} l\right) f(x)dx \\ &\quad + \int_0^1 (1-x)U(-\alpha^*(C,T) - T) f(x)dx = pU(-l-T) + (1-p)U(-T),\end{aligned} \tag{A3-2}$$

where $\overline{x}_1 = \frac{1}{l}[\alpha^*(C,T) + C + \overline{K} - M]$.

The insurer's optimal insurance premium $\alpha^e(T)$ and optimal capital $C^e(T)$ given any T and $M = E[I_{re}(x)] - T$ are proved under different market structures. In a monopolistic market where $\eta = 1$, the insurer decides the insurance premium α and its capital C to maximize its expected profit, i.e., $\max_{\alpha,C} \Pi(\alpha, C, R^e = 1, M) = \int_0^{\overline{x}} [\alpha - M - xl + I_{re}(x)] f(x)dx - [1 - F(\overline{x})]C - rC$. Given any C, the insurer's expected profit $\Pi(\alpha, C, R^e = 1, M)$ is increasing in α and thus the insurer sets the premium at individuals' maximum willingness to pay $\alpha^*(C,T)$. The optimal capital $C^*(T)$ given any T satisfies:

$$\frac{d}{dC}\Pi(\alpha, C, R^e = 1, M) = F(\overline{x}^*(T)) \frac{d}{dC}\alpha^*(C^*(T), T) - [1 - F(\overline{x}^*(T)) + r] = 0, \tag{A3-3}$$

where $\overline{x}^*(T) = \frac{1}{l}[\alpha^*(C^*(T), T) + C^*(T) + \overline{K} - E[I_{re}(x)] + T]$. Thus, the optimal insurance premium $\alpha^*(T)$ given any T is $\alpha^*(T) = \alpha^*(C^*(T), T)$. Therefore, the monopolistic insurer's expected profit given any T and $M = E[I_{re}(x)] - T$ is:

$$\Pi^*(M,T) = \int_0^{\bar{x}^*(T)} [\alpha^*(T) - M - xl + I_{re}(x)] f(x)dx - [1 - F(\bar{x}^*(T))]C^*(T) - rC^*(T). \quad \text{(A3-4)}$$

In a perfectly competitive market where $\eta = 0$, the insurer's optimal capital $C^{**}(T)$ given any T is derived from the zero expected profit condition:

$$\int_0^{\bar{x}^{**}(T)} [\widetilde{\alpha} - E[I_{re}(x)] + T - xl + I_{re}(x)] f(x)dx - [1 - F(\bar{x}^{**}(T))]C^{**}(T) - rC^{**}(T) = 0. \quad \text{(A3-5)}$$

In markets with imperfect competition where $0 < \eta < 1$, the insurer decides the premium α and its capital C to maximize its expected profit with two constraints: (i) the limited market pricing power constraint, i.e., $\alpha = \eta\alpha^*(C,T) + (1-\eta)\widetilde{\alpha}$ and (ii) the profitability constraint, i.e., $\Pi(\alpha, C, R^e = 1, M) \leq \eta\Pi^*(M,T)$. In the equilibrium, the optimal capital $C^e(T;\eta)$ given any T is the sum of the optimal premium income capital and the optimal product quality capital, i.e., $C^e(T;\eta) = C^e_{quality}(T;\eta) + C^e_{premium}(T;\eta)$. The optimal premium income capital $C^e_{premium}(T;\eta)$ given T maximizes the insurer's expected profit without considering the profitability constraint:

$$\eta F(\bar{x}_2(T)) \frac{d}{dC} \alpha^*(C^e_{premium}(T;\eta), T) - [1 - F(\bar{x}_2(T)) + r] = 0, \quad \text{(A3-6)}$$

where $\bar{x}_2(T) = \frac{1}{l}[\alpha^*(C^e_{premium}(T;\eta), T) + C^e_{premium}(T;\eta) + \overline{K} - M]$. The optimal product quality capital $C^e_{quality}(T;\eta)$ given T can then be derived from the profitability constraint:

$$\int_0^{\bar{x}^e(T)} [\alpha^e(T) - M - xl + I_{re}(x)] f(x)dx - [1 - F(\bar{x}^e(T)) + r][C^e_{quality}(T;\eta) + C^e_{premium}(T;\eta)] = \eta\Pi^*(M,T), \quad \text{(A3-7)}$$

where $\bar{x}^e(T) = \frac{1}{l}[\alpha^e(T) + C^e(T;\eta) + \overline{K} - M]$. The optimal insurance premium $\alpha^e(T;\eta)$ is at the weighted sum of individuals' maximum willingness to pay and the exogenous market price, i.e., $\alpha^e(T;\eta) = \eta\alpha^*(C^e(T;\eta), T) + (1-\eta)\widetilde{\alpha}$. When $\eta = 1$ ($\eta = 0$), we have $\alpha^e(T;1) = \alpha^*(T)$ ($\alpha^e(T;0) = \widetilde{\alpha}$) and $C^e(T;1) = C^*(T)$ ($C^e(T;0) = C^{**}(T)$).

Government's Decisions

The government decides the catastrophe tax T and the reinsurance premium M to maximize the expected social utility with some constraints. The budget constraint of the government $T + M \geq E[I_{re}(x)]$ is binding in the equilibrium. If the government reinsurance has a surplus, the government can always improve the expected social utility by lowering the reinsurance premium or the catastrophe tax.

The insurer's non-negative profit constraint, i.e., $\Pi(\alpha^e(T;\eta), C^e(T;\eta), R^e = 1, M) \geq 0$ is always satisfied given any T and $M = E[I_{re}(x)] - T$. Given any $T \geq 0$, the government reinsurance premium is lower than the private reinsurance premium, i.e., $M \leq M_p$. *Ceteris paribus*, the insurer's expected profit is higher with lower reinsurance premium. Thus, the insurer's expected profit in the GR equilibrium is higher than that in the PR equilibrium. Given the insurer's expected profit in the PR equilibrium is non-negative, the insurer's expected profit is always non-negative in the GR case with any $T \geq 0$.

Thus, the optimization problem of the government can be simplified as $\max_T V(T) = V\left(\alpha^e(T;\eta), C^e(T;\eta), R^e = 1, E[I_{re}(x)] - T, T\right)$. The optimal catastrophe tax T^e can be derived from the first order condition:

$$\frac{d}{dT}V(T^e) = \frac{d}{dT}U_i\left(\alpha^e(T^e;\eta), T^e\right) + \frac{d}{dT}\Pi\left(\alpha^e(T^e;\eta), C^e(T^e;\eta), R^e = 1, M^e\right) = 0. \qquad \text{(A3-8)}$$

The optimal reinsurance premium can then be derived from the binding budget constraint:

$$M^e = E[I_{re}(x)] - T^e. \qquad \text{(A3-9)}$$

Recall that given any catastrophe tax T, the insurer's optimal premium is $\alpha^e(T;\eta)$ and its optimal capital is $C^e(T;\eta)$. Given the optimal catastrophe tax T^e, we have the the insurer's optimal decisions in the GR equilibrium as shown in Eq.(12)-Eq.(17).

Therefore, each individual's expected utility, the insurer's expected profit, and the expected social utility are as follows:

$$U_i^e = \int_0^{\bar{x}^e} xU(-\alpha^e(\eta) - T)f(x)dx + \int_{\bar{x}^e}^1 xU(-\alpha^e(\eta) - T - l + \frac{\bar{x}^e}{x}l)f(x)dx + \int_0^1 (1-x)U(-\alpha^e(\eta) - T)f(x)dx, \qquad \text{(A3-10)}$$

$$\Pi^e = \eta\Pi^*(M^e, T^e), \qquad \text{(A3-11)}$$

$$V^e = U_i^e + \Pi^e. \qquad \text{(A3-12)}$$

Appendix A4: Proof of Equilibrium with Alternative Social Welfare Functions

Consider that the social expected utility is defined as the weighted sum of individuals' expected utilities and the insurer's expected profit, i.e., $\hat{V}(\alpha, C, R, M, T) \equiv \omega U_i(\alpha, T) + (1-\omega)\Pi(\alpha, C, R, M)$, where $0 \leq \omega \leq 1$. We prove the equilibrium with this weighted social welfare function with backward induction. Given any catastrophe tax T and reinsurance premium M, individuals' and the insurer's optimal decisions are proved in Appendix A3. All individuals buy the catastrophe insurance, i.e., $\hat{\lambda}^e = 1$. The insurer buys the government reinsurance, i.e., $\hat{R}^e = 1$, sets the primary insurance premium at $\alpha^e(T;\eta)$, and holds capital at $C^e(T;\eta) = C^e_{quality}(T;\eta) + C^e_{premium}(T;\eta)$. Last, we come the government's optimal decisions.

With individuals' and the insurer's optimal decisions as well as the government's binding budget constraint, the weighted expected social utility is

$$\begin{aligned}\hat{V}(T) &\equiv \hat{V}(\alpha^e(T;\eta), C^e(T;\eta), R^e = 1, E[I_{re}(x)] - T, T) \\ &= \omega U_i(\alpha^e(T;\eta), T) + (1-\omega)\Pi(\alpha^e(T;\eta), C^e(T;\eta), R^e = 1, E[I_{re}(x)] - T).\end{aligned} \tag{A4-1}$$

Let $\frac{d}{dT}\hat{V}(\hat{T}(\eta)) = 0$. Thus, if $\hat{T}(\eta)$ satisfies $0 \le \hat{T} \le E[I_{re}(x)]$, $\hat{T}^e(\eta) = \hat{T}(\eta)$ is the optimal catastrophe tax. Under different market structures η and the weight of individuals' expected utilities ω, the optimal catastrophe tax may not be interior.

In a perfect competitive market where $\eta = 0$, the insurer's expected profit is zero and thus maximizing the weighted expected social utility is equivalent to maximizing individuals' expected utilities for the government. The weight ω does not affect the government's optimal decisions, i.e.,

$$\hat{T}^e(\eta = 0) = \hat{T}(\eta = 0). \tag{A4-2}$$

In markets with imperfect competition or a monopolistic market where $0 < \eta \le 1$, the insurer's expected profit is increasing in the catastrophe tax and thus $\forall T$, $\frac{d\hat{V}(T)}{dT} > 0$ if $\omega = 0$. There exists $0 < \underline{\omega}(\eta) < 1$ such that $\frac{d}{dT}\hat{V}\left(E[L_{re}(x)]\right) = 0$. If $\omega = 1$ and $\eta = 1$, individuals' uninsured expected utilities are decreasing in the catastrophe tax and thus $\forall T$, $\frac{d\hat{V}(T)}{dT} < 0$. Therefore, there exists η_1 such that when $\eta_1 < \eta \le 1$, $\forall T$, $\frac{d\hat{V}(T)<0}{dT}$. Thus, there exists $0 < \overline{\omega}(\eta) < 1$ $(\eta_1 < \eta \le 1)$ such that $\frac{d}{dT}\hat{V}(0) = 0$. Thus, the optimal catastrophe tax is

$$\hat{T}^e(\eta) = \begin{cases} E[L_{re}(x)], & \text{if } 0 \le \omega < \underline{\omega}(\eta) \\ \hat{T}(\eta), & \text{if } \underline{\omega}(\eta) \le \omega \le \overline{\omega}(\eta) \\ 0, & \text{if } \overline{\omega}(\eta) < \omega \le 1, \end{cases} \tag{A4-3}$$

where $0 < \overline{\omega}(\eta) < 1$ when $\eta_1 < \eta \le 1$ and $\overline{\omega}(\eta) = 1$ when $0 < \eta \le \eta_1$. When $\eta = 0$, $\underline{\omega}(\eta) = 0$ and $\overline{\omega}(\eta) = 1$. The optimal reinsurance premium is derived from the binding budget constraint, i.e., $\hat{M}^e(\eta) = E[I_{re}(x)] - \hat{T}^e(\eta)$. Recall that given any catastrophe tax T, the insurer's optimal insurance premium is $\alpha^e(T;\eta)$ and its optimal capital is $C^e(T;\eta)$. Given the optimal catastrophe tax $\hat{T}^e(\eta)$ in the equilibrium, the optimal insurance premium is $\hat{\alpha}^e(\eta) = \alpha^e(\hat{T}^e(\eta);\eta)$ and the insurer's optimal capital is $\hat{C}^e(\eta) = C^e(\hat{T}^e(\eta);\eta)$.

With the weighted social welfare function, our propositions still hold if $\hat{T}^e(\eta) > 0$. The detailed proof is similar to that with an unweighted social welfare function, which is shown in Appendix B. If $\hat{T}^e(\eta) = 0$, the GR equilibrium is equivalent to the PR equilibrium.

Appendix B: Proof of Propositions

Appendix B1: Proof of Proposition 1

In the NR case, when the risk correlation is high, i.e., $\delta \to 1$, almost all individuals suffer a loss l simultaneously with probability p. Thus, given any C_n,

$$\alpha_n^*(C_n) \to -U^{-1}\left(\frac{p(U(-l)-U(C_n-l))}{1-p}\right), \text{ as } \delta \to 1, \tag{B1-1}$$

$$\underline{\alpha}_n(C_n) \to \frac{p+r}{1-p}C_n, \text{ as } \delta \to 1, \tag{B1-2}$$

where individuals' maximum willingness to pay $\alpha_n^*(C_n)$ is independent of r and the insurer's minimum acceptable premium $\underline{\alpha}_n(C_n)$ is increasing to infinity in r. Thus, there exists a minimum capital cost $\underline{r}$ such that when $r > \underline{r}$ and $\delta \to 1$, $\alpha_n^*(C_n^e) < \underline{\alpha}_n(C_n^e)$. In other words, given $r > \underline{r}$, there exists a risk correlation threshold $\underline{\delta}$ such that when $\delta > \underline{\delta}$, $\alpha_n^*(C_n^e) < \underline{\alpha}_n(C_n^e)$ and the catastrophe insurance market fails to exist.

In the PR case, when the insurer holds zero capital, its expected profit is positive when the premium is set at individuals' maximum willingness to pay $\alpha_p^*(0)$:

$$\begin{aligned}\Pi(\alpha_p^*(0), C_p = 0, R_p = 1) &= \int_0^{\bar{x}_p} [\alpha_p^*(0) - M_p - xl + I_{re}(x)] f(x) dx \\ &= \alpha_p^*(0) - M_p - \int_0^{\bar{x}_p} [xl - I_{re}(x)] f(x) dx - \int_{\bar{x}_p}^1 [\alpha_p^*(0) - M_p] f(x) dx \\ &> E[xI(x)] - M_p - \int_0^{\bar{x}_p} [xl - I_{re}(x)] f(x) dx - \int_{\bar{x}_p}^1 [\alpha^*(0) - M_p] f(x) dx \\ &= 0,\end{aligned} \tag{B1-3}$$

where the inequality $\alpha_p^*(0) > E[xI(x)]$ follows from individuals' risk averse. Given the optimal capital C_p^e, the insurer's expected profit with the premium at individuals' maximum willingness to pay $\alpha_p^*(C_p^e)$ is strictly higher than that with the minimum acceptable premium $\underline{\alpha}_p(C_p^e)$:

$$\begin{aligned}\Pi(\alpha_p^*(C_p^e), C_p^e, R_p = 1) &\geq \Pi(\eta\alpha_p^*(C_p^e) + (1-\eta)\widetilde{\alpha}, C_p^e, R_p = 1) \\ &= \eta\Pi_p^* \\ &\geq \eta\Pi(\alpha_p^*(0), C_p = 0, R_p = 1) \\ &\geq 0 \\ &= \Pi(\underline{\alpha}_p(C_p^e), C_p^e, R_p = 1),\end{aligned} \tag{B1-4}$$

where the first inequality follows from that the insurer's expected profit is increasing in the insurance premium and the equal sign holds if and only if $\eta = 1$; the first equality is the insurer's binding profitability constraint; the second inequality follows from that Π_p^* is the maximum expected profit of a monopolistic insurer, which is the highest expected profit that the insurer is able to earn; the last inequality follows from Eq.(B1-3) and the equal sign holds if and only if $\eta = 0$; the last equality follows from that the insurer's expected profit is zero at minimum acceptable premium $\underline{\alpha}_p(C_p^e)$. Given that the equal signs of the first and last inequality do not hold simultaneously, we have $\Pi(\alpha_p^*(C_p^e), C_p^e, R_p = 1) > \Pi(\underline{\alpha}_p(C_p^e), C_p^e, R_p = 1)$ and thus $\alpha_p^*(C_p) > \underline{\alpha}_p(C_p)$.

Similarly, in the GR case, we can prove that the insurer's expected profit with the premium at individuals' maximum willingness to pay $\alpha^*(C^e, T^e)$ is strictly higher than that with the premium at the insurer's minimum acceptable premium $\underline{\alpha}(C^e, M^e)$, i.e., $\Pi(\alpha^*(C^e, T^e), C^e, R = 1, M^e) > \Pi(\underline{\alpha}(C^e, M^e), C^e, R = 1, M^e)$ and $\alpha^*(C^e, T^e) > \underline{\alpha}(C^e, M^e)$.

Appendix B2: Proof of Proposition 2

The proof is in two steps: (i) private reinsurance improves expected social utility compared with the equilibrium with no reinsurance. (ii) government reinsurance improves expected social utility compared with the equilibrium with private reinsurance and achieves Pareto improvement in some competitive markets.

Step (i): Comparing Eqs.(A2-2,A2-3) with Eqs.(A1-5,A1-6), we have $\alpha_n^* = \alpha_p^*$ and $C_n^* = C_p^* + \overline{K} - M_p$. Thus, in a monopolistic market,

$$\begin{aligned}
\Pi_p^* &= \int_0^{\bar{x}_p^*} \left[\alpha_p^* - M_p - xl + I_{re}(x)\right] f(x)dx - [1 - F(\bar{x}_p^*)]C_p^* - rC_p^* \\
&= \int_0^{\bar{x}_n^*} (\alpha_n^* - xl) f(x)dx - [1 - F(\bar{x}_n^*)]C_n^* - rC_n^* + r(\overline{K} - M_p) \\
&> \Pi_n^*,
\end{aligned} \tag{B2-1}$$

where the inequality follows from $\overline{K} - M_p > 0$.According to Eq.(A1-13) and Eq.(A2-9), the insurer's expected profit is $\Pi_n^e = \eta \Pi_n^*$ and $\Pi_p^e = \eta \Pi_p^*$ in the NR and PR equilibriums, respectively. Therefore, when $\eta = 0$, $\Pi_n^e = \Pi_p^e = 0$, and when $0 < \eta \leq 1$, together with Eq.(B2-1), $\Pi_n^e < \Pi_p^e$. Therefore, we have $\Pi_n^e \leq \Pi_p^e$.

Denote $U_{ip}(C_p)$ each individual's expected utility in the PR case when the insurer holds capital C_p and sets the premium at $\eta \alpha_p^*(C_p) + (1 - \eta)\widetilde{\alpha}$. We can prove that when the insurer holds capital at $C_p = C_n^e - (\overline{K} - M_p)$ in the PR case, each individual's expected utility equals to that in the NR equilibrium, $U_{ip}(C_n^e - (\overline{K} - M_p)) = U_{in}^e$ and the insurer's expected profit is higher than $\eta \Pi_p^*$:

$$\Pi_p(\alpha_p(\eta), C_n^e - (\overline{K} - M_p), R_p^e = 1) = \eta \Pi_p^* + (1 - \eta) r(\overline{K} - M_p) \geq \eta \Pi_p^*. \tag{B2-2}$$

Required by the profitability constraint, the insurer should earn an expected profit no more than $\eta\Pi_p^*$ and thus should hold capital more than $C_n^e-(\overline{K}-M_p)$:

$$C_p^e \geq C_n^e-(\overline{K}-M_p), \text{ and the equal sign holds iff } \eta=1. \tag{B2-3}$$

Each individual's expected utility in the PR case $U_{ip}(C_p)$ is increasing in C_p:

$$\frac{dU_{ip}(C_p)}{dC_p}=\eta\frac{\partial U_{ip}(C_p)}{\partial \alpha_p(\eta)}\frac{\partial \alpha_p^*(C_p)}{\partial C_p}+\frac{\partial U_{ip}(C_p)}{\partial C_p}\geq 0, \text{ and the equal sign holds iff } \eta=1. \tag{B2-4}$$

In the PR equilibrium, each individual's expected utility is $U_{ip}^e=U_{ip}(C_p^e)$. Given Eqs.(B2-2,B2-4,B2-5), each individual's expected utility in the NR equilibrium is lower than that in the PR equilibrium, i.e., $U_{in}^e\leq U_{ip}^e$ (strictly holds when $0\leq\eta<1$).

In conclusion, $\Pi_n^e\leq\Pi_p^e$ (strictly holds when $0<\eta\leq 1$), $U_{in}^e\leq U_{ip}^e$ (strictly holds when $0\leq\eta<1$) and thus $V_n^e<V_p^e$.

Step (ii): We compare the GR and PR equilibriums. The expected social utility in the PR equilibrium equals to that in the GR case when $T=0$, i.e., $V_p^e=V(0)$. Since $T^e=argmax_T V(T)>0$, $V(0)<V(T^e)$ and $V_p^e<V^e$.

According to Eq.(A2-9) and Eq.(A3-17), in the PR and GR equilibriums, the insurer's expected profit is $\Pi_p^e=\eta\Pi_p^*$ and $\Pi^e=\eta\Pi^*(M^e,T^e)$, respectively. Therefore when $\eta=0$, $\Pi_p^e=\Pi^e=0$. When $0<\eta\leq 1$, together with $\Pi_p^*<\Pi^*(M^e,T^e)$, we have $\Pi_p^e<\Pi^e$. Overall, the insurer's expected profit in the PR equilibrium is lower than that in the GR equilibrium, i.e., $\Pi_p^e\leq\Pi^e$.

Next, we compare individuals' expected utilities in the PR and GR equilibriums. In a perfectly competitive market where $\eta=0$, $\Pi_p^{**}=\Pi^{**}=0$ and thus $V_p^e=U_{ip}^e$ and $V^e=U_i^e$. Recall that $V_p^e<V^e$, and thus

$$U_{ip}^e<U_i^e \text{ when } \eta=0. \tag{B2-5}$$

In a monopolistic market where $\eta=1$, individuals end up with uninsured expected utility in the PR and GR equilibriums, i.e. $U_{ip}^e=pU(-l)$ and $U_i^e=pU(-T^e-l)+(1-p)U(-T^e)$. Because $pU(-l)>pU(-T^e-l)+(1-p)U(-T^e)$, we have

$$U_{ip}^e>U_i^e \text{ when } \eta=1. \tag{B2-6}$$

Given Eq.(B2-5) and Eq.(B2-6) and according to the continuity of η, there exists $\overline{\eta}$ such that when $0\leq\eta\leq\overline{\eta}$, $U_{ip}^*\leq U_i^*$ and the Pareto improvement is achieved. When $\overline{\eta}<\eta\leq 1$, $U_{ip}^*>U_i^*$.

Appendix B3: Proof of Proposition 3

First we compare the insurer's default threshold in the NR and PR equilibrium. Comparing Eqs.(A2-2,A2-3) with Eqs.(A1-5,A1-6), we have $\alpha_n^* = \alpha_p^*$ and $C_n^* = C_p^* + \overline{K} - M_p$. Thus, in a monopolistic market where $\eta = 1$, $\overline{x}_p^* = \frac{1}{l}[\alpha_p^* + C_p^* + \overline{K} - M_p] = \frac{1}{l}(\alpha_n^* + C_n^*) = \overline{x}_n^*$. In competitive markets where $0 \le \eta < 1$,

$$\begin{aligned} \overline{x}_p^e &= \frac{1}{l}[\eta\alpha_p^*(C_p^e(\eta)) + (1-\eta)\widetilde{\alpha} + C_p^e(\eta) + \overline{K} - M_p] \\ &> \frac{1}{l}[\eta\alpha_n^*(C_n^e(\eta)) + (1-\eta)\widetilde{\alpha} + C_n^e(\eta)] \\ &= \overline{x}_n^e, \end{aligned} \quad \text{(B3-1)}$$

where the inequality follows from Eq.(B2-3) and $\alpha_p^*(C_p^e(\eta))$ is increasing in $C_p^e(\eta)$.

Then we compare the insurer's default threshold in the PR and GR equilibrium. In a perfectly competitive market where $\eta = 0$, the insurer's default threshold given any catastrophe tax T is $\overline{x}^{**}(T) = \frac{1}{l}[\widetilde{\alpha} + C^{**}(T) + \overline{K} - E[I_{re}(x)] + T]$. Taking derivative with respect to T, we have

$$\frac{d\overline{x}^{**}(T)}{dT} = \frac{dC^{**}(T)}{dT} + 1 > 0, \text{i.e., } \overline{x}_p^{**} < \overline{x}^{**}. \quad \text{(B3-2)}$$

In a monopolistic market where $\eta = 1$, the insurer's default threshold given any catastrophe tax T is $\overline{x}^*(T) = \frac{1}{l}[\alpha^*(C^*(T),T) + C^*(T) + \overline{K} - E[I_{re}(x)] + T]$. In the PR (GR) equilibrium, the insurer's default threshold is $\overline{x}_p^* = \overline{x}^*(0)$ ($\overline{x}^* = \overline{x}^*(T^*)$). Taking derivative with respect to T, we have, if individuals exhibit CARA,

$$\frac{d\overline{x}^*(T)}{dT} = \frac{\partial\alpha^*(C^*(T),T)}{\partial T} + \frac{\partial\alpha^*(C^*(T),T)}{\partial C} \times \frac{\partial C^*(T)}{\partial T} + \frac{\partial C^*(T)}{\partial T} + 1 \quad = 0, \quad \text{(B3-3)}$$

where $\frac{\partial\alpha^*(C^*(T),T)}{\partial T} = \frac{\partial\alpha^*(C^*(T),T)}{\partial C}$ and $\frac{\partial C^*(T)}{\partial T} = -1$. Thus, if individuals exhibit CARA preference,we have $\overline{x}_p^* = \overline{x}^*$ .Similarly, we can prove that $\overline{x}_p^* < \overline{x}^*$ if individuals exhibit DARA preference and $\overline{x}_p^* > \overline{x}^*$ if individuals exhibit IARA preference.

Finally, according to the continuity of η, if individuals exhibit DARA or CARA, $\forall 0 \le \eta \le 1$, $\overline{x}_p^e \le \overline{x}^e$. If individuals exhibit IARA, there exists $0 < \widetilde{\eta} < 1$ such that when $0 \le \eta < \widetilde{\eta}$, $\overline{x}_p^e < \overline{x}^e$ and when $\widetilde{\eta} \le \eta \le 1, \overline{x}_p^e \ge \overline{x}^e$.

Appendix C: Proof of Impact Channels and Pricing of Government Reinsurance

Appendix C1: Proof of Product Quality Channel

Recall that the insurer's optimal product quality capital $C^e_{quality}(T;\eta)$ given any T is derived from the binding profitability constraint driven by the competitive pressure:

$$\int_0^{\bar{x}^e} [\alpha^e(T) - M - xl + I_{re}(x)] f(x)dx - [1 - F(\bar{x}^e) + r][C^e_{quality}(T;\eta) + C^e_{premium}(T;\eta)] = \eta\Pi^*(M,T). \tag{A3-7}$$

In the PR and GR equilibriums, the insurer's optimal product quality capital is $C^e_{quality,p}(\eta) = C^e_{quality}(T=0;\eta)$ and $C^e_{quality}(\eta) = C^e_{quality}(T=T^e;\eta)$, respectively.

In a monopolistic market where $\eta = 1$, the insurer is not subject to a profitability constraint and thus holds zero product quality capital, i.e., $C^e_{quality,p}(\eta=1) = C^e_{quality}(\eta=1) = 0$.

In a competitive market where $0 \le \eta < 1$, the insurer in the GR case holds the product quality capital at the level of the PR equilibrium, i.e., holding capital $\hat{C} = C^e_{quality,p}(\eta) + C^e_{premium}(\eta)$, it will earn an expected profit higher than $\eta\Pi^*(M^e,T^e)$:

$$\begin{aligned} &\Pi(\alpha(\eta),\hat{C},R^e=1,M^e) - \eta\Pi^*(M^e,T^e) \\ &> \int_0^{T^e} \left(\eta F(\hat{\bar{x}})\frac{\partial}{\partial C}\alpha^*(\hat{C},T) - [1-F(\hat{\bar{x}})] - r\right)\frac{\partial}{\partial T}C^e_{premium}(T;\eta)dT \\ &>0, \end{aligned} \tag{C1-1}$$

where $\hat{\bar{x}} = \frac{1}{l}[\alpha^*(\hat{C},T) + \hat{C} + \overline{K} - M]$. $\Pi(\alpha(\eta),\hat{C},R=1,M^e) > \eta\Pi^*(M^e,T^e)$ follows from that $\eta F(\hat{\bar{x}})\frac{\partial}{\partial C}\alpha^*(\hat{C},T) - [1-F(\hat{\bar{x}})] - r < 0$ and $\frac{\partial}{\partial T}C^e_{premium}(T;\eta) < 0$. Together with the profitability constraint, i.e., $\Pi(\alpha(\eta),C^e(\eta),R^e=1,M^e) = \eta\Pi^*(M^e,T^e)$, we have

$$\Pi(\alpha(\eta),\hat{C},R^e=1,M^e) > \Pi(\alpha(\eta),C^e(\eta),R^e=1,M^e), \tag{C1-2}$$

where $\hat{C} > C^e_{premium}(\eta)$, $C^e(\eta) > C^e_{premium}(\eta)$. Since $\hat{C} = C^e_{quality,p}(\eta) + C^e_{premium}(\eta) \ge C^e_{premium}(\eta)$ and the insurer's expected profit $\Pi(\alpha(\eta),C,R=1,M^e)$ is decreasing in capital C when $C \ge C^e_{premium}(\eta)$, we have $\hat{C} < C^e(\eta)$. Recall that $\hat{C} = C^e_{quality,p}(\eta) + C^e_{premium}(\eta)$ and $C^e(\eta) = C^e_{quality}(\eta) + C^e_{premium}(\eta)$, and thus we have $C^e_{quality,p}(\eta) < C^e_{quality}(\eta)$.

Appendix C2: Proof of Capital Cost Channel

Recall that the insurer's optimal premium income capital $C^e_{premium}(T;\eta)$ given any T is the capital that maximizes the insurer's expected profit without considering the profitability constraint:

$$\begin{aligned}&\frac{d}{dC}\Pi(\eta\alpha^*(C^e_{premium}(T;\eta),T)+(1-\eta)\widetilde{\alpha},C^e_{premium}(T;\eta),R^e=1,M)\\&=\eta F(\hat{x})\frac{d}{dC}\alpha^*(C^e_{premium}(T;\eta),T)-[1-F(\hat{x})+r]=0.\end{aligned}\tag{C2-1}$$

In a perfectly competitive market where $\eta=0$, the marginal benefits of holding premium income capital is zero, i.e., $\eta F(\hat{x})\frac{d}{dC}\alpha^*(C^e_{premium}(T;\eta),T)=0$ but its marginal cost is positive, i.e., $1-F(\hat{x})+r>0$. Therefore, the insurer holds zero premium income capital in the perfectly competitive market, i.e., $C^e_{premium,p}(\eta)=C^e_{premium}(\eta)=0$ when $\eta=0$.

In a market with imperfect competition and in a monopolistic market where $0<\eta\leq 1$, according to the Implicit Function Theorem, taking derivative with respect to T in Eq.(C2-1) yields

$$\frac{dC^e_{premium}(T;\eta)}{dT}=-\frac{\frac{d^2}{dCdT}\Pi(\alpha^*(C^e_{premium}(T;\eta),T),T,C^e_{premium}(T;\eta),R^e=1,M)}{\frac{d^2}{dC^2}\Pi(\alpha^*(C^e_{premium}(T;\eta),T),T,C^e_{premium}(T;\eta),R^e=1,M)}<0,\tag{C2-2}$$

where the denominator $\frac{d^2\Pi}{dC^2}<0$ follows from that $C^e_{premium}(T;\eta)$ maximizes the insurer's expected profit and the numerator

$$\frac{d^2\Pi}{dCdT}=\frac{d}{d\alpha^*(C^e_{premium}(T;\eta),T)}\frac{d\Pi}{dC}\times\frac{\partial\alpha^*(C^e_{premium}(T;\eta),T)}{\partial T}+\frac{\partial}{\partial T}\frac{d\Pi}{dC}<0.\tag{C2-3}$$

In the PR and GR equilibriums, the insurer's optimal premium income capital is $C^e_{premium,p}(\eta)=C^e_{premium}(T=0;\eta)$ and $C^e_{premium}(\eta)=C^e_{premium}(T=T^e;\eta)$, respectively. Given Eq.(C2-2) and $T^e>0$, we have $C^e_{premium,p}(\eta)<C^e_{premium}(\eta)$ when $0<\eta\leq 1$.

Appendix C3: Proof of Pricing of Government Reinsurance

First, we prove that the government reinsurance is risk-based. According to the government's binding budget constraint, we have $M^e+T^e=E[I_{re}(x)]=\int_{\frac{K}{l}}^{\frac{K+\overline{K}}{l}}(xl-K)f(x)dx+\int_{\frac{K+\overline{K}}{l}}^{1}\overline{K}f(x)dx$. With some algebra, we have

$$\frac{\partial(M^e+T^e)}{\partial l}=\int_{\frac{K}{l}}^{\frac{K+\overline{K}}{l}}xf(x)dx>0,\tag{C3-1}$$

$$\frac{\partial(M^e+T^e)}{\partial K}=F\left(\frac{K}{l}\right)-F\left(\frac{K+\overline{K}}{l}\right)<0,\tag{C3-2}$$

$$\frac{\partial(M^e+T^e)}{\partial\overline{K}}=1-F\left(\frac{K+\overline{K}}{l}\right)>0.\tag{C3-3}$$

Next, we prove that the government reinsurance is affordable, i.e., $\alpha^e-M^e>0$:

$$0\leq\Pi^e=\int_0^{\overline{x}^e}[\alpha^e-M^e-xl+I_{re}(x)]f(x)dx-[1-F(\overline{x}^e)]C^e-rC^e<\int_0^{\overline{x}^e}(\alpha^e-M^e)f(x)dx.\tag{C3-4}$$

Next, we prove that the government reinsurance is long-term sustainable with a cost loading $r' \geq 0$. Assuming that the share of catastrophe-hit population in each period $x_1, x_2, ..., x_t$ are independent and identically distributed, $I_{re}(x_1), I_{re}(x_2), ..., I_{re}(x_t)$ are also independent and identically distributed. With the Law of Large Numbers, if $T' + M' = (1+r')E[I_{re}(x_t)]$, then

$$\frac{1}{t}\sum_{1}^{t}[T' + M' - (1+r')I_{re}(x_t)] \rightarrow T' + M' - (1+r')E[I_{re}(x_t)] = 0, \text{ as } t \rightarrow \infty. \tag{C3-5}$$

If the government budget is ex-ante binding in each period, i.e., $T' + M' = (1+r')E[I_{re}(x_t)]$, the government reinsurance can break even and be sustainable in t periods (as t is sufficiently large) ex post.

Consider a loan interest rate $r_0 > 0$ and a reserve investment return rate $r_1 > 0$, according to the Law of Large Numbers, the t-period average surplus of the government reinsurance is

$$\begin{aligned}
&\frac{1}{t}\sum_{1}^{t}\{(1+r_0)\min\{0, [T + M - I_{re}(x_t)]\} + (1+r_1)\max\{0, [T + M - I_{re}(x_t)]\}\} \\
=&\frac{1}{t}\sum_{1}^{t}\{(1+r_1)[T + M - I_{re}(x_t)] + (r_0 - r_1)\min\{0, [T + M - I_{re}(x_t)]\}\} \\
\rightarrow&(1+r_1)\{T + M - E[I_{re}(x)]\} + (r_0 - r_1)E\left[\min\{0, [T + M - I_{re}(x_t)]\}\right] \text{ as } t \rightarrow \infty,
\end{aligned} \tag{C3-6}$$

where $E\left[\min\{0, [T + M - I_{re}(x_t)]\}\right] < 0$ and $T + M - E[I_{re}(x)] = 0$ following from the binding government budget.

If $r_0 < r_1$, the government reinsurance earns a surplus in t periods, i.e., $(r_0 - r_1)E[\min\{0, [T + M - I_{re}(x_t)]\}] > 0$. Thus, without the concern about the long-term sustainability, the government could ease the burden of individuals and the insurer by charging a reinsurance price lower than the expected loss, i.e., $T + M < E[I_{re}(x)]$.

If $r_1 = r_0$, the government reinsurance breaks even in t periods, i.e., $(r_0 - r_1)E[\min\{0, [T + M - I_{re}(x_t)]\}] = 0$. The government reinsurance program is long-term sustainable.

If $r_1 < r_0$, the government reinsurance would have a deficit in t periods, i.e., , $(r_0 - r_1)E[T + M - I_{re}(x)|I_{re}(x) \geq T + M] < 0$. However, there exists a $r' > 0$ such that

$$T + M - E[I_{re}(x)] + \frac{r_0 - r_1}{1 + r_1}E[T + M - I_{re}(x)|I_{re}(x) \geq T + M] = T + M - (1+r')E[I_{re}(x)], \tag{C3-7}$$

the left hand side of Eq.(C3-7) is independent of r' and the right hand side is decreasing in r', strictly larger than left hand side when $r' = 0$. In other words, if the investment return rate r_1 is lower than the interest rate r_0, it is equivalent to have a cost loading r' in each period, which we have proved its sustainability.

Appendix D: Proof and Additional Results for COVID-19 Applications

Appendix D1: Additional Calibration Results for COVID-19

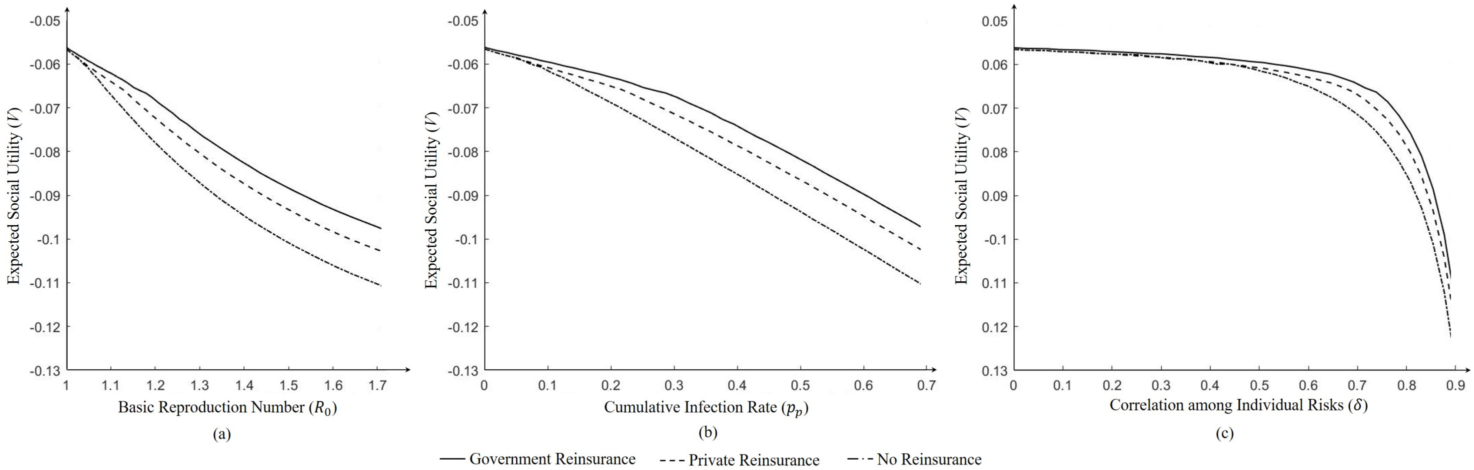

Figure D1-1: COVID-19 and Expected Social Utility when $\eta = 0.75$

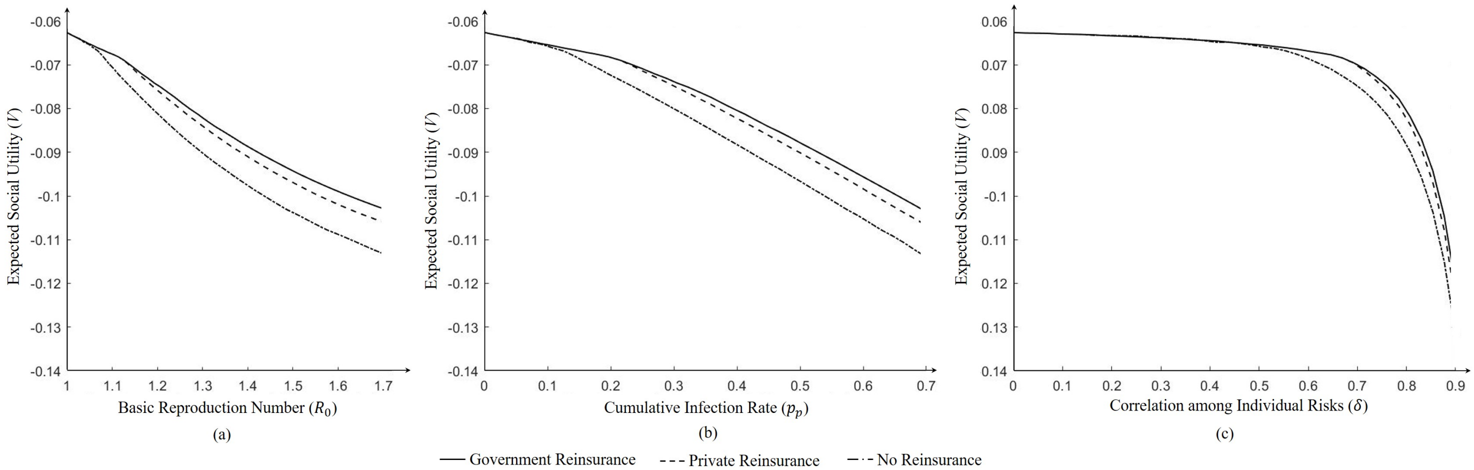

Figure D1-2: COVID-19 and Expected Social Utility when $\eta = 0.5$

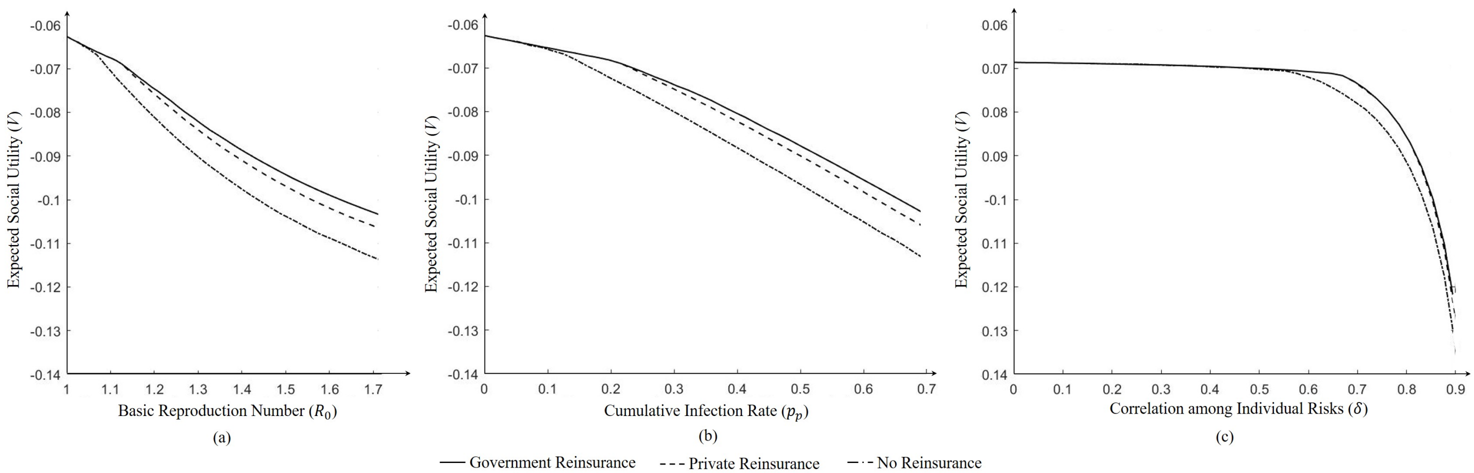

Figure D1-3: COVID-19 and Expected Social Utility when $\eta = 0.25$

Appendix D2: Proof of Corollary 1

Corollary 1.1

We solve the equilibrium using backward induction and start from period 2. In period 2, the three decision-makers repeat the single-period game as shown in Appendix A3. Since there is no cost for open decision such as intensifying pandemic , it is always better for businesses to open and buy pandemic insurance to share risks in period 2. We denote the period-2 expected utility in the equilibrium after open (close) in period 1 as $U_{2i}(q)$ ($U_{2i}(p)$) .

In period 1, we derive the decisions of the three decision-makers in the equilibrium with backward induction and start from businesses. If businesses choose to open in period 1, it is easy to prove that buying pandemic insurance brings them higher expected utility and businesses will thus buy pandemic insurance. Given any catastrophe tax T_1, primary premium α_1 and the insurer's capital C_1, the period-1 expected utility with pandemic insurance is

$$U_{1i}(q) = \int_0^{\bar{x}_q} xU\left(-\alpha_1 - T_1\right) f(x)dx + \int_{\bar{x}_q}^1 xU\left(-\alpha_1 - T_1 - l + \frac{\bar{x}_q}{x} l\right) f(x)dx + \int_0^1 (1-x)U(-\alpha_1 - T_1) f(x)dx. \tag{D2-1}$$

If businesses choose to close in period 1, they bear the losses l themselves and thus the period-1 expected utility is $U_{1i} = U(-l - T_1)$. It is easy to prove that period-1 expected utility with pandemic insurance is higher than that without insurance, i.e., $U_{1i}(q) > U_{1i}$ for any $T_1 \geq 0$, $0 < \alpha_1 < l$, $C_1 \geq 0$ and $q < 1$. But whether business open and buy insurance in period 1 or close in period 1 depends on inter-temporal expected utility, the sum of period-1 expected utility and period-2 expected utility, which is related to inter-temporal correlation q. Next, we consider two extreme cases where $q = p$ and $q = 1$.

With zero inter-temporal correlation, i.e., $q = p$, businesses' period-2 expected utility is independent with their decisions in period 1, i.e., $U_{2i}(q) = U_{2i}(p)$. Thus, businesses only need to consider their period-1 expected utility when making decisions. Businesses have higher period-1 expected utility if they choose to open and buy the insurance, i.e., $U_{1i}(q) > U_{1i}$, and thus in the equilibrium when $q = p$, businesses choose to open and buy the insurance in period 1 and achieve higher inter-temporal expected utility, i.e.,

$$U_{1i}(q) + U_{2i}(q) > U_{1i} + U_{2i}(p), \text{ when } q = p. \tag{D2-2}$$

With highest inter-temporal correlation, i.e., $q = 1$, the pandemic will cause a certain losses l to businesses in each period once they open, i.e., $ql = [q^2 + (1-q)p]l = l$. Given any catastrophe tax

T_1, primary premium α_1 and the insurer's capital C_1, if businesses choose to open in period 1,their period-1 expected utility is

$$U_{1i}(q) = U(-l-\alpha_1-T_1+\bar{x}_q l) = U(-l+C_1), \tag{D2-3}$$

where $\bar{x}_q l = \alpha_1 + C_1 + \overline{K} - M_1$ and $M_1 + T_1 = E[I_{re}(x)] = \overline{K}$. The insurer's expected profit is $-(1+r)C_1$ because the loss l is certain to occur and the insurer pays out all its capital. Thus, in the equilibrium, the insurer will hold zero capital, i.e., $C_1 = 0$ and businesses' period-1 expected utility becomes $U_{1i}(q) = U(-l)$. Since businesses' expected utility and the insurer's expected profit are both independent with T_1, in the equilibrium, the optimal catastrophe tax in period 1 follows $T_1 = 0$ which yields $U_{1i} = U(-l-T_1) = U(-l)$. In sum, we have $U_{1i}(q) = U_{1i} = U(-l)$ and $U_{2i}(q) < U_{2i}(p)$. Thus, in the equilibrium when $q = 1$, it is better for businesses to close in period 1, i.e.,

$$U_{1i}(q) + U_{2i}(q) < U_{1i} + U_{2i}(p), \text{ when } q = 1. \tag{D2-4}$$

Given Eq.(D2-2) and Eq. (D2-4), moreover $U_{1i}(q) + U_{2i}(q)$ decreases with q and q is continuous, there exists $\hat{q}$ such that in the equilibrium when $p \le q \le \hat{q}$, businesses choose to open and buy pandemic insurance in period 1, i.e., $U_{1i}(q) + U_{2i}(q) \ge U_{1i} + U_{2i}(p)$ and when $\hat{q} < q \le 1$, businesses choose to close in period 1, i.e., $U_{1i}(q) + U_{2i}(q) < U_{1i} + U_{2i}(p)$.

Corollary 1.2

As shown in the proof of Corollary 1.1, we only need to consider two possible paths: (1) businesses keep opening and buy pandemic insurance in both periods; (2) businesses close without pandemic insurance in period 1, but open and buy pandemic insurance in period 2. We denote them as Path 1 and Path 2 hereafter.

To prove corollary 1.2, we first make some definitions.We denote the catastrophe tax in period 1 and 2 as T_1 and T_2 respectively. In both periods, the reinsurance coverage is $\overline{K}$. In Path 1, we denote the insurer's expected profit in the equlibrium in period 1 and period 2 given any T_1, T_2 and $\overline{K}$ as $\Pi_1(T_1, \overline{K}, q)$ and $\Pi_2(T_2, \overline{K}, q)$ respectively; and denote businesses' expected utility in the equilibrium in period 1 and 2 given any T_1, T_2 and $\overline{K}$ as $U_1(T_1, \overline{K}, q)$ and $U_2(T_2, \overline{K}, q)$ respectively. In Path 2, the insurer's period-1 expected profit is 0 and businesses' period-1 expected utility is $U(-l-T_1)$ given any T_1. We denote the insurer's period-2 expected profit as $\Pi_2(T_2, \overline{K}, p)$ and businesses' period-2 expected utility as $U_2(T_2, \overline{K}, p)$, the same part in Path 1 and 2. We denote the threshold of inter-temporal correlation for market existence in period 1 given any T_1, T_2 and $\overline{K}$ as

$\hat{q}(T_1,T_2,\overline{K})$. Hence, $\hat{q}_p = \hat{q}(0,0,\overline{K})$ stands for the threshold in the private reinsurance. Note that $\hat{q}(T_1,T_2,\overline{K})$ makes businesses indifferent between Path 1 and 2, i.e.,

$$U_{1i}(T_1,\overline{K},\hat{q}(T_1,T_2,\overline{K})) + U_{2i}(T_2,\overline{K},\hat{q}(T_1,T_2,\overline{K})) = U(-l-T_1) + U_{2i}(T_2,\overline{K},p), \tag{D2-5}$$

And the insurer is indifferent between Path 1 and 2 with $\hat{q}(T_1,T_2,\overline{K})$:

$$\Pi_1(T_1,\overline{K},\hat{q}(T_1,T_2,\overline{K})) + \Pi_2(T_2,\overline{K},\hat{q}(T_1,T_2,\overline{K})) = \Pi_2(T_2,\overline{K},p). \tag{D2-6}$$

Then we prove that $\frac{\partial \hat{q}_p}{\partial \overline{K}} > 0$ and $\frac{\partial \hat{q}}{\partial \overline{K}} > 0$ in a monopolistic market where $\eta = 1$. Businesses' period-2 expected utility in Path 1 and 2 are $U_{2i}(T_2,\overline{K},q) = [q^2 + (1-q)p]U(-l-T_2) + [1-q^2-(1-q)p]U(-T_2)$ and $U_{2i}(T_2,\overline{K},p) = pU(-l-T_2) + (1-p)U(-T_2)$, both of which are independent with $\overline{K}$. Thus, for any $\overline{K}_a < \overline{K}_b$, we have

$$U_{2i}(T_2,\overline{K}_a,q) = U_{2i}(T_2,\overline{K}_b,q) \text{ and } U_{2i}(T_2,\overline{K}_a,p) = U_{2i}(T_2,\overline{K}_b,p). \tag{D2-7}$$

In period 2, the monopolistic insurer use the net coverage of reinsurance as one-to-one substitute to it capital, i.e., $\frac{\partial C_2(T_2,\overline{K})}{\partial(\overline{K}-E[I_{re}(x)])} = -1$, which is derived from Eq. (A3-3) by taking derivative with respect to $\overline{K} - E[I_{re}(x)]$. Thus, we have

$$\frac{\partial \Pi_2(T_2,\overline{K},q)}{\partial \overline{K}} = \frac{\partial \Pi_2(T_2,\overline{K},q)}{\partial(\overline{K}-E[I_{re}(x)])} \times \frac{\partial(\overline{K}-E[I_{re}(x)])}{\partial \overline{K}} = r \times \frac{\partial(\overline{K}-E[I_{re}(x)])}{\partial \overline{K}}. \tag{D2-8}$$

Similarly, we have $\frac{\partial \Pi_2(T_2,\overline{K},p)}{\partial \overline{K}} = r \times \frac{\partial(\overline{K}-E[I_{re}(x)])}{\partial \overline{K}}$. In turn, we have $\frac{\partial \Pi_2(T_2,\overline{K},q) - \partial \Pi_2(T_2,\overline{K},p)}{\partial \overline{K}} = 0$. Thus, for any $\overline{K}_a < \overline{K}_b$, we have

$$\Pi_2(T_2,\overline{K}_a,q) - \Pi_2(T_2,\overline{K}_a,p) = \Pi_2(T_2,\overline{K}_b,q) - \Pi_2(T_2,\overline{K}_b,p). \tag{D2-9}$$

In period 1, given any T_1 and $\overline{K}$, the monopolistic insurer set the premium at businesses' maximum willingness to pay $\alpha_1^*(C_1,T_1,\overline{K})$ such that businesses are indifferent between Path 1 and 2, and holds optimal capital $C_1(T_1,\overline{K})$ to maximize its expected profit. Thus, when $p \le q \le \hat{q}(T_1,T_2,\overline{K})$, we have

$$\begin{aligned} U_{1i}(T_1,\overline{K},q) + [q^2 + (1-q)p]U(-l-T_2) + [1-q^2-(1-q)p]U(-T_2) \\ = U(-l-T_1) + pU(-l-T_2) + (1-p)U(-T_2), \end{aligned} \tag{D2-10}$$

$$F(\bar{x}_q)\frac{\partial}{\partial C_1}\alpha_1^*(C_1(T_1,\overline{K}),T_1,\overline{K}) = 1 - F(\bar{x}_q) + r, \tag{D2-11}$$

where

$$\begin{aligned} U_{1i}(T_1,\overline{K},q)=&\int_0^{\bar{x}_q} xU\left(-\alpha_1^*(C_1(T_1,\overline{K}),T_1,\overline{K})-T_1\right)f(x)dx \\ &+\int_{\bar{x}_q}^1 xU\Big(-\alpha_1^*(C_1(T_1,\overline{K}),T_1,\overline{K})-T_1-l+\frac{\bar{x}_q}{x}l\Big)f(x)dx \\ &+\int_0^1(1-x)U(-\alpha_1^*(C_1(T_1,\overline{K}),T_1,\overline{K})-T_1)f(x)dx. \end{aligned} \tag{D2-12}$$

The monopolistic insurer's period-1 expected profit is

$$\Pi_1(T_1,\overline{K},q)=\int_0^{\bar{x}_q}[\alpha_1^*(C_1(T_1,\overline{K}),T_1,\overline{K})+T_1-E[I_{re}(x)]-xl+I_{re}(x)]f(x)dx-[1-F(\bar{x}_q)+r]C_1(T_1,\overline{K},q). \tag{D2-13}$$

Taking derivative with respect to $\overline{K}$ and together with $\frac{\partial C_1(T_1,\overline{K})}{\partial(\overline{K}-E[I_{re}(x)])}=-1$, $\frac{\partial\alpha_1^*(C_1(T_1,\overline{K}),T_1,\overline{K})}{\partial C_1(T_1,\overline{K})}=\frac{\partial\alpha_1^*(C_1(T_1,\overline{K}),T_1,\overline{K})}{\partial(\overline{K}-E[I_{re}(x)])}$, and $\frac{\partial U_{1i}(T_1,\overline{K},q)}{\partial C_1(T_1,\overline{K})}=\frac{\partial U_{1i}(T_1,\overline{K},q)}{\partial(\overline{K}-E[I_{re}(x)])}$ and with some algebra, we have

$$\frac{\partial\Pi_1(T_1,\overline{K},q)}{\partial\overline{K}}=r\times\frac{\partial(\overline{K}-E[I_{re}(x)])}{\partial\overline{K}}>0, \tag{D2-14}$$

$$\frac{\partial U_{1i}(T_1,\overline{K},q)}{\partial\overline{K}}=0. \tag{D2-15}$$

For any given $\overline{K}_a<\overline{K}_b$, according to Eq.(D2-14) and Eq.(D2-15), we have

$$\Pi_1(T_1,\overline{K}_a,q)<\Pi_1(T_1,\overline{K}_b,q)\text{ and }U_{1i}(T_1,\overline{K}_a,q)=U_{1i}(T_1,\overline{K}_b,q). \tag{D2-16}$$

Therefore, given $\overline{K}_b$, $\overline{K}_a<\overline{K}_b$ and $q=\hat{q}(T_1,T_2,\overline{K}_a)$, the insurer earns higher expected profit in Path 1:

$$\begin{aligned} &\Pi_1(T_1,\overline{K}_b,\hat{q}(T_1,T_2,\overline{K}_a))+\Pi_2(T_2,\overline{K}_b,\hat{q}(T_1,T_2,\overline{K}_a))-0-\Pi_2(T_2,\overline{K}_b,p) \\ =&\Pi_1(T_1,\overline{K}_b,\hat{q}(T_1,T_2,\overline{K}_a))+\Pi_2(T_2,\overline{K}_a,\hat{q}(T_1,T_2,\overline{K}_a))-\Pi_2(T_2,\overline{K}_a,p) \\ >&\Pi_1(T_1,\overline{K}_a,\hat{q}(T_1,T_2,\overline{K}_a))+\Pi_2(T_2,\overline{K}_a,\hat{q}(T_1,T_2,\overline{K}_a))-\Pi_2(T_2,\overline{K}_a,p) \\ =&0, \end{aligned} \tag{D2-17}$$

where the first equality follows Eq.(D2-9), the first inequality follows Eq.(D2-16), and the last equality follows Eq.(D2-6). And individuals are indifferent between Path 1 and 2:

$$\begin{aligned} &U_{1i}(T_1,\overline{K}_b,\hat{q}(T_1,T_2,\overline{K}_a))+U_{2i}(T_2,\overline{K}_b,\hat{q}(T_1,T_2,\overline{K}_a))-U(-l-T_1)-U_{2i}(T_2,\overline{K}_b,p) \\ =&U_{1i}(T_1,\overline{K}_a,\hat{q}(T_1,T_2,\overline{K}_a))+U_{2i}(T_2,\overline{K}_a,\hat{q}(T_1,T_2,\overline{K}_a))-U(-l-T_1)-U_{2i}(T_2,\overline{K}_a,p) \\ =&0, \end{aligned} \tag{D2-18}$$

where the first equality follows Eq.(D2-7) and Eq.(D2-16), and the second equality follows Eq.(D2-5). Since $\hat{q}(T_1,T_2,\overline{K}_b)$ should satisfy Eq.(D2-5) and Eq.(D2-6), we have $\hat{q}(T_1,T_2,\overline{K}_a)<\hat{q}(T_1,T_2,\overline{K}_b)$. That is, for any given $\overline{K}_a<\overline{K}_b$, we have $\hat{q}(T_1,T_2,\overline{K}_a)<\hat{q}(T_1,T_2,\overline{K}_b)$ when $\eta=1$. Equivalently, we have $\frac{\partial \hat{q}_p}{\partial \overline{K}} > 0$.

Next, we prove that $\hat{q}_p > \hat{q}$ in a monopolistic market where $\eta = 1$. In the government reinsurance case when $q = \hat{q}$, the government's optimal tax in period 1 follows $T_1 = 0$, because the expected social utility decreases with T_1, i.e.,

$$\begin{aligned} V_1 + V_2 &= U_{1i}(T_1, \overline{K}, \hat{q}(T_1, T_2, \overline{K})) + U_{2i}(T_2, \overline{K}, \hat{q}(T_1, T_2, \overline{K})) + \Pi_1(T_1, \overline{K}, \hat{q}(T_1, T_2, \overline{K})) + \Pi_2(T_2, \overline{K}, \hat{q}(T_1, T_2, \overline{K})) \\ &= U(-l - T_1) + U_{2i}(T_2, \overline{K}, p) + \Pi_2(T_2, \overline{K}, p) \end{aligned} \tag{D2-19}$$

decreases with T_1, where the equality follows from Eq.(D2-5) and Eq.(D2-6).$T_1 = 0$ definitely holds in the private reinsurance case. Thus, we compare $\hat{q}_p$ and $\hat{q}$ by focusing on T_2 only. Businesses' period-2 expected utility in Path 1 and 2 are $U_{2i}(T_2, \overline{K}, q) = [q^2 + (1-q)p]U(-l - T_2) + [1 - q^2 - (1-q)p]U(-T_2)$ and $U_{2i}(T_2, \overline{K}, p) = pU(-l - T_2) + (1-p)U(-T_2)$ respectively.Taking derivative with T_2, we have

$$\frac{\partial [U_{2i}(T_2, \overline{K}, q) - U_{2i}(T_2, \overline{K}, p)]}{\partial T_2} = -q(q-p)U'(-l - T_2) - [1 - q(q-p)]U'(-T_2) < 0, \tag{D2-20}$$

and thus, given ant $T_2 > 0$,

$$U_{2i}(T_2, \overline{K}, q) - U_{2i}(T_2, \overline{K}, p) < U_{2i}(0, \overline{K}, q) - U_{2i}(0, \overline{K}, p). \tag{D2-21}$$

Thus, given any $T_2 > 0$ and $q = \hat{q}_p$, it is better for businesses to close proactively in period 1, i.e.,

$$\begin{aligned} &U_{1i}(0, \overline{K}, \hat{q}_p) + U_{2i}(T_2, \overline{K}, \hat{q}_p) - U(-l) - U_{2i}(T_2, \overline{K}, p) \\ <&U_{1i}(0, \overline{K}, \hat{q}_p) + U_{2i}(0, \overline{K}, \hat{q}_p) - U(-l) - U_{2i}(0, \overline{K}, p) \\ =&0, \end{aligned} \tag{D2-22}$$

where the inequality follows from Eq.(D2-21).That is ,the market with government reinsurance cannot survive with the threshold of private reinsurance case. Thus, the upper threshold for market existence in the government reinsurance case $\hat{q}$ is lower than $\hat{q}_p$ in a monopolistic market when $\eta = 1$.

In conclusion, when $\eta = 1$, we have $\frac{\partial \hat{q}_p}{\partial \overline{K}} > 0$, $\frac{\partial \hat{q}}{\partial \overline{K}} > 0$, and $\hat{q}_p > \hat{q}$. Thus, there must exist an interval $\acute{\eta} < \eta \leq 1$ such that $\frac{\partial \hat{q}_p}{\partial \overline{K}} > 0$, $\frac{\partial \hat{q}}{\partial \overline{K}} > 0$ and $\hat{q}_p > \hat{q}$ still hold.

Corollary 1.3

The proof is similar to the proof of Proposition 2 in Appendix B2.

Corollary 1.4

We first consider the condition where the market exists under moral hazard. Given any premium α_1, the insurer's capital C_1 and tax T_1 in period 1, if businesses buy pandemic insurance and then close proactively in period 1, their period-1 expected utility follows $U(-\alpha_1 - T_1 - l + \overline{x}_q l) = U(-l + C_1)$. If businesses open with pandemic insurance, the period-1 expected utility is $U_{1i}(T_1, \overline{K}, q) + U_{2i}(T_2, \overline{K}, \hat{q})$. Note that the pandemic insurance market fails if businesses close proactively but with insurance because of the insurer's default. Thus, the market exists when it is better for businesses with pandemic insurance to open rather than closing proactively in period 1, i.e.,

$$U_{1i}(T_1, \overline{K}, q) + U_{2i}(T_2, \overline{K}, \hat{q}) > U(-l + C_1) + U_{2i}(T_2, \overline{K}, p), \tag{D2-23}$$

Meanwhile, it is better for the insurer to sell pandemic insurance rather than exit the market in period 1, i.e.,

$$\Pi_1(T_1, \overline{K}, q) + \Pi_2(T_2, \overline{K}, \hat{q}) > \Pi_2(T_2, \overline{K}, p). \tag{D2-24}$$

With the conditions above, the market exists in the absence of moral hazard because businesses achieve higher expected utility from opening in period 1, i.e.,

$$\begin{aligned} U_{1i}(T_1, \overline{K}, q) + U_{2i}(T_2, \overline{K}, \hat{q}) &\geq U(-l + C_1) + U_{2i}(T_2, \overline{K}, p) \\ &> U(-l) + U_{2i}(T_2, \overline{K}, p), \end{aligned} \tag{D2-25}$$

and the insurer achieves higher expected profit from opening in period 1 as Eq.(D2-24) holds. Thus, we have $[p, \hat{q}_{mh}] \subset [p, \hat{q}]$, equivalently, $\hat{q}_{mh} < \hat{q}$.

Appendix D3: Proof of Corollary 2

Corollary 2.1

First, the proof for PRIA is shown in Appendix B1. For BCPP, the minimum acceptable premium is $\underline{\alpha}_{BCPP} = pl$. Since business are risk averse, their maximum willingness to pay is higher than the expected losses (the minimum acceptable premium), i.e., $\alpha^*_{BCPP} > pl = \underline{\alpha}_{BCPP}$.

Corollary 2.2

In a perfectly competitive market where $\eta = 0$, the insurer's expected profit is zero, i.e., $\Pi^e_{PRIA} = 0$. Since BCPP offers the insurance in highest quality (i.e., zero default probability) and charges at fair premium, individuals has higher expected utility in the BCPP case, i.e., $U^e_{PRIA} <$

U^e_{BCPP}. Thus, we have

$$V^e_{PRIA} - V^e_{BCPP} < 0, \text{ when } \eta = 0. \tag{D3-1}$$

In a monopolistic market where $\eta = 1$, we have

$$V^e_{PRIA} - V^e_{BCPP} > 0, \text{ when } \eta = 1, \tag{D3-2}$$

which follows from $\left(V^e_{PRIA} - V^e_{BCPP}\right)|_{p\to 0} = 0$ and $\frac{\partial\left(V^e_{PRIA} - V^e_{BCPP}\right)}{\partial p}|_{p\to 0} > 0$. Given Eq.(D3-1) and Eq.(D3-2) and according to the continuity of η, there exists $0 \le \hat{\eta} \le 0$ such that $V^e_{PRIA} \le V^e_{BCPP}$ when $0 \le \eta \le \hat{\eta}$ and $V^e_{PRIA} > V^e_{BCPP}$ when $\hat{\eta} < \eta \le 1$.

Corollary 2.3

Given $p < \frac{K}{tl}$, government insurance faces higher deficit probability:

$$\begin{aligned} DP^e_{BCPP} = Prob(\sum_{i=1}^{t} x_i l > tpl) > Prob(\sum_{i=1}^{t} x_i l > K) > Prob(\sum_{i=1}^{t} I_{re}(x_i) l > 0) > Prob\left(\sum_{i=1}^{t} I_{re}(x_i) l > tE[I_{re}(x)]\right) \\ = DP^e_{PRIA}, \end{aligned} \tag{D3-3}$$

where the first inequality follows from $p < \frac{K}{tl}$.Given that government faces a deficit, i.e., $\sum_{i=1}^{t} x_i l > tpl$ ($\sum_{i=1}^{t} I_{re}(x_i) l > tE[I_{re}(x)]$), the government insurance faces higher deficit severity:

$$\begin{aligned} DS^e_{BCPP} = \sum_{i=1}^{t} x_i l - tpl > \sum_{i=1}^{t} x_i l - K > \sum_{i=1}^{t} I_{re}(x_i) l - tE[I_{re}(x)] \\ = DS^e_{PRIA}, \end{aligned} \tag{D3-4}$$

where the first inequality follows from $p < \frac{K}{tl}$ and the second inequality follows from $\sum_{i=1}^{t} I_{re}(x_i) l > tE[I_{re}(x)]$.

Corollary 2.4

First we consider the case of BCPP. Supposing $q > p$, in period 2, businesses will open and buy the government insurance in the equilibrium, the government's budget constraint is binding and thus the government insurance is delivered in fair price. Businesses' period-2 expected utility is $U(-[q^2 + (1-q)p]l)$ when they open in period 1 and $U(-pl)$ when they close proactively in period 1. Given any premium of BCPP in period 1 α_{BCPP1} less than loss l, Path 2 brings higher expected utility to businesses than Path 1 and thus it is always better for businesses to close in period 1 with $q > p$, i.e.,

$$U(-\alpha_{BCPP1}) + U(-[q^2 + (1-q)p]l) < U(-\alpha_{BCPP1}) + U(-pl), \ \forall q > p. \tag{D3-5}$$

But if that is the case, a certain loss l happens and the government's budget can not break even in period 1 with $\alpha_{BCPP1} < l$. However,the government budget constraint must be binding in the equilibrium and thus we have $\hat{q}_{BCPP} = p$.

Then we consider the case of PRIA when $q = p$. Since $q = p$, we have $q^2 + (1-q)p = p$, i.e., businesses' period-2 expected utility and the insurer's period-2 expected profit are independent with businesses' opening decision in period 1. Thus, both businesses and the insurer consider their period-1 expected utility/profit when making decisions.The inter-temporal problem degenerates into our single-period risk-sharing model. As proved in the equilibrium of single-period model in Appendix A3, the pandemic insurance market always exists with government reinsurance, i.e., Path 2 brings higher expected utility to businesses and higher expected profit to the insurer than Path 1. Thus, we have $\hat{q}_{PRIA} > p$. Together with $\hat{q}_{BCPP} = p$, we have $\hat{q}_{BCPP} < \hat{q}_{PRIA}$.

Appendix E: Proof of Alternative Government Intervention Policies

Appendix E1: Proof of the RR Case

First, we prove that in a monopolistic market, the insurer always holds zero capital in the RR case, i.e., $C_{rr}^* = 0$. Individuals' maximum willingness to pay $\alpha_{rr}^*(T_{rr})$ satisfies

$$U(-\alpha_{rr}^*(T_{rr}) - T_{rr}) = pU(-T_{rr} - l) + (1-p)U(-T_{rr}), \tag{E1-1}$$

suggesting that $\alpha_{rr}^*(T_{rr})$ is independent of the insurer's capital C_{rr}, i.e., $\frac{\partial \alpha_{rr}^*(T_{rr})}{\partial C_{rr}} = 0$. $C_{rr}^* = 0$ follows from that holding capital has zero marginal benefit, i.e., $F(\bar{x}_{rr})\frac{\partial \alpha_{rr}^*(T_{rr})}{\partial C_{rr}} = 0$, but has a positive marginal cost, i.e., $1 - F(\bar{x}_{rr}) + r > 0$.

Next, we prove $\alpha_{CL}^* < \alpha_{rr}^*(0)$ where $\alpha_{rr}^*(0)$ is individuals' maximum willingness to pay when $T_{rr} = 0$:

$$U(-\alpha_{rr}^*(0)) = pU(-l) = \int_0^1 U(-\alpha_{CL}^* - T(x))f(x)dx < U[E(-\alpha_{CL}^* - T(x))] = U[-\alpha_{CL}^* - E(T(x))]. \tag{E1-2}$$

Given $U'(\cdot) > 0$, we have $\alpha_{CL}^* < \alpha_{rr}^*(0)$.

Next, we prove $V_{rr}^e > V_{CL}^e$. Following Charpentier and Le Maux's (2014) setup for the catastrophe relief program (the GRel case), we compare V_{rr}^e and V_{CL}^e in a monopolistic market. In the GRel equilibrium, the insurer holds an exogenous capital $C_{CL} > 0$ and sets the insurance premium at individuals' maximum willingness to pay α_{CL}^*. Because $C_{rr}^* = 0 < C_{CL}$ and $\alpha_{rr}^*(0) > \alpha_{CL}^*$, the insurer in the RR case with $T_{rr} = 0$ earns a higher expected profit than that in the GRel equilibrium. In a monopolistic market, individuals' expected utilities is $pU(-l)$ in both the RR case with $T_{rr} = 0$ and the GRel equilibrium. Thus, $V_{rr}(0) > V_{CL}^e$, where $V_{rr}(0)$ is the expected social utility in the RR case with $T_{rr} = 0$. Since V_{rr}^e is the optimized expected social utility in the RR equilibrium, we have $V_{rr}^e \geq V_{rr}(0) > V_{CL}^e$.

Finally, we prove $V_{rr}^e > V^e$. Denote $\hat{V}_{rr}$, $\hat{\Pi}_{rr}$, and $\hat{U}_{rr}$ as the expected social utility, the insurer's expected profit, and each individual's expected utility in the RR case when $\alpha_{rr} = \alpha^e$, $C_{rr} = C^e$ and $M_{rr} = M^e$. With the same primary catastrophe insurance premium, capital, and reinsurance premium, the insurer's expected profit is the same in the RR and GR equilibriums, i.e., $\hat{\Pi}_{rr} = \Pi^e$. In the RR case, the budget constraint of the government is

$$T_{rr} + M_{rr} \geq E[I_{re}(x)] + \int_{\bar{x}}^1 (x - \bar{x}) l f(x)dx. \tag{E1-3}$$

When $M_{rr} = M^e$, to meet the above budget constraint, the government need to charge higher catastrophe tax from individuals to cover the default loss, i.e., $T_{rr} = T^e + \int_{\bar{x}}^1 (x - \bar{x}) l f(x)dx$. In the

GR case, individuals bear the default loss while in the RR case, the government covers the default loss by charging the additional tax $T_{rr} - T^e$, which is the fair value of the default loss. Risk-averse individuals are better off by transferring the default loss to the government at the fair price, i.e., $\hat{U}_{rr} > U_i^e$. Thus, the expected social utility in the RR case when $\alpha_{rr} = \alpha^e$, $C_{rr} = C^e$ and $M_{rr} = M^e$ is higher than that in the GR equilibrium, i.e., $\hat{V}_{rr} > V^e$. Since V_{rr}^e is the maximum expected social utility in the RR case, we have $V_{rr}^e \geq \hat{V}_{rr} > V^e$.

Appendix E2: Proof of Solvency Regulation

It is straightforward that $V^e = V_s^e(\theta)$ when the maximum acceptable default probability of the solvency regulation is larger than the insurer's default probability in the unregulated GR equilibrium, i.e., $\theta \geq \theta^e$.

When $\theta < \theta^e$, for $\forall\ \theta_2 < \theta_1 < \theta^e$, we denote T_{si}^e and M_{si}^e $(i = 1, 2)$ as the optimal catastrophe tax and reinsurance premium, and C_{si}^e as the optimal capital of the insurer with θ_i as the maximum acceptable default probability of the solvency regulation. According to Eq.(29), we have $C_{s2}^e > C_{s1}^e > C^e$. Recall that holding capital more than the optimal level C^e in the unregulated GR case decreases the insurer's expected profit. Thus, assuming the same reinsurance premium M_{s2}^e in both Case 1 and Case 2, the insurer's expected profit in Case 1 with capital C_{s1}^e is higher than that in the Case 2 with capital C_{s2}^e:

$$\Pi(C_{s1}^e, M_{s2}^e; \theta_1) > \Pi(C_{s2}^e, M_{s2}^e; \theta_2) = \eta \Pi^*(M_{s2}^e, T_{s2}^e), \tag{E2-1}$$

where the equality is the binding profitability constraint in Case 2. If the government charges the reinsurance premium at M_{s2}^e in Case 1, the insurer's expected profit will violate the profitability constraint in Case 1. To lower the insurer's expected profit such that the profitability constraint is satisfied, the reinsurance premium in Case 1 should be higher than that in Case 2, i.e., $M_{s1}^e > M_{s2}^e$. Given the binding budget constraint of the government, we have $T_{s1}^e < T_{s2}^e$, i.e., $\frac{\partial T_s^e}{\partial \theta} < 0, \forall \theta \leq \theta^e$. Since $\frac{\partial V_s^e(\theta)}{\partial \theta} = \frac{dV(T)}{dT}|_{T=T_s^e} \times \frac{\partial T_s^e}{\partial \theta}$ and $\frac{dV(T)}{dT} < 0$ when $T > T^e$, we have $\frac{\partial V_s^e(\theta)}{\partial \theta} > 0$.